

PUPIL AND TEACHER PERCEPTIONS OF
THE TAXONOMIC CLASSIFICATION OF SENIOR
CERTIFICATE MATHEMATICS HIGHER GRADE
EXAMINATION QUESTIONS

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A dissertation submitted to the Faculty of Education,
University of Cape Town, in partial fulfilment of
the requirements for the degree of Master of
Education.

1983

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Thanks

I should like to express my thanks to all the teachers who so willingly co-operated with the collection of the data for this dissertation. In addition, I should also like to thank the education authorities of both the Cape Education Department and the Department of Internal Affairs who allowed me such ready access into the schools and afforded me such helpful co-operation. To Kevin Rochford of the University of Cape Town whose guidance was of inestimable worth my grateful thanks. Finally to my wife my heartfelt thanks for all her encouragement and patience.

Abstract

This dissertation seeks to apply a taxonomic classification to Senior Certificate Mathematics examination questions. Candidates who wrote a typical question paper as well as the teachers who taught them were asked to give their taxonomic perceptions of each question. The taxonomic classifications of both pupils and teachers are compared. The effect of the ability of pupils in the subject on their classifications is considered and a relationship between performance score and pupil classification is shown to exist.

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Chapter 1

Origin and Aims of the Present Research

Origin of the Research and Statement of the Problem

In paper I written by the present writer on a critical approach to the use of Bloom's and other taxonomies for evaluation in Mathematics, it was stated (p.62) that there was a worldwide trend of accepting the usefulness of taxonomies without backing this acceptance with empirical findings. As noted there, it is only recently that empirical studies have been carried out. This trend is also very evident in South African research publications.

It appears that far too little research into examining is being conducted at the secondary school level - and this in a country where the Senior Certificate examination is almost revered. It is easy to appreciate why many researchers in education, who are generally university-based, use university students for their empirical investigations - they are readily available and permission does not have to be sought from other authorities in order to be able to carry out the research. However, one is left with the feeling that there is a gap which needs to be filled because there is a tendency to apply what has been learnt at the tertiary level to the secondary, without the support of empirical validation for such a downward transfer.

Furthermore, if one wishes to use a taxonomy, one of the major considerations is to establish which questions are to be classified as Knowledge and which as testing higher order abilities. To ascertain this information the best person to ask is most probably the student. Hence it was decided to ascertain student perceptions of questions in a traditional Senior Certificate Mathe-

matics Higher Grade examination paper and to see how their perceptions accord with those of the teachers teaching them and to see whether there is any relationship between these perceptions and the students' performance scores.

Suggestions for research given in Paper I

Bloom's taxonomy and others, such as that of Avital and Shettleworth which are especially geared for evaluation in Mathematics, are intended for the use of teachers. Published research on the taxonomies, as far as the writer is aware, has been confined to a consideration of questions whose taxonomic classification has been predetermined by the researcher. This raises the question as to whether pupils will give the same taxonomic classification to questions as their teachers do. Hence the following suggestions for empirical work were made in paper I:-

1. "It would be profitable to ascertain pupil opinion on the questions in an examination paper, especially with regard to whether they have answered that type of question before." (p.62). Further to this, it would be interesting to see whether in each class group the assessment of the pupil accords with that of the teacher, and whether there is overall agreement between teachers and pupils on the taxonomic classification of examination questions in mathematics.
2. "It would be of use to compare on what type of questions pupils who obtained high, average or low marks gained their marks." It might be expected that above average pupils will gain high marks on questions of all taxonomic orders, whereas below average pupils will tend to gain high marks on questions classified in the knowledge category only.

3. "A question-by-question analysis of marks gained at the various hierarchical levels to test whether the hierarchy applies would be worthwhile." One of the problems that has to be faced is that the taxonomies have generally been applied to multiple-choice questions, whereas in South Africa mathematics question papers are generally not set in this form. It is not as easy to decide whether a question is of a particular type when a discursive answer is required, as a number of additional factors are introduced into each answer. In mathematics, however, this is not as serious a problem as it would be in other subjects because, by and large, a specific type of answer is required. Thus the process of deciding into which classification each of the questions falls is made only a little more subjective than it would be otherwise.
4. "There needs to be an agreement as to the proportion of marks that should be allocated at each hierarchical level. In order to do this the judgments of teachers and tertiary education lecturers should be sought." (p.63)

Aims of the Empirical work in this Dissertation

This final dissertation describes the empirical research carried out by the present writer in order to attempt to find answers to the first three problems raised above.

More specifically the questions investigated are as follows:-

1. Does the taxonomic classification of questions in mathematics examination papers by pupils differ *significantly* from that by teachers?
2. Is the taxonomic classification of questions by

pupils significantly affected by their ability:

- (i) in the subject as a whole; and
- (ii) to answer any given question?

3. Do pupils of higher ability in a subject agree with teachers' taxonomic classifications to a more significant degree than those of lower ability?
4. Do pupils mean scores on mathematics examination papers decrease as the taxonomic classification increases (Kropp et al, 1966)? Both pupils' and teachers' classifications must be considered.

Chapter 2

Theoretical Bases of the Investigation

The Taxonomic Basis of the Study

As outlined in Essay I (pp 11 - 23) Bloom's classification as a taxonomic basis for mathematics has shortcomings, so it has been decided to use the modification first suggested by Avital and Shettleworth (1968) with a further higher order ability of "Inventiveness" added on, as suggested by Wood (1968a). Thus the taxonomic classes (with the abbreviations used in this dissertation) are:

1. Knowledge (K)

This classification is given to questions of a type to which pupils have been exposed before, and in which the algorithmic processes are simple.

2. Comprehension and Application (CA)

This classification is given to questions of a type to which students have been exposed before, and in which the algorithmic processes are more complex, thus requiring an application of the knowledge learnt along with a need to comprehend fully what is entailed in the problem. Because the algorithmic processes are more complex it is held that there is no straightforward recall of what has been learnt.

The above two categories are subdivisions of the major category : questions to which students have been exposed before. The following major category comprises questions to which students have not been exposed before. It is also sub-divided into two parts.

3. Analysis and Synthesis (AS)

This classification is given to questions to which the students have not been exposed before, but

in which they can see the connection between their knowledge and the new situation.

4. Inventiveness (I)

This classification is given to questions which ask something completely different from what the pupil is used to or has previously encountered.

The Need for Empirical Evidence for the Theoretical Taxonomies

As stated in essay I (pp 52 and 53) it would certainly seem that far too few experiments have been carried out on Bloom's and related taxonomies, and it would appear that his findings have had far too ready acceptance without their being tested. It would also appear that in testing taxonomies in different subject areas, researchers have tended to avoid mathematics as nearly all the empirical findings concern themselves with the behavioural and natural sciences (Kibler 1974 and Winne 1979). This is despite Wood's confident assertion in 1968:

"Although Bloom's taxonomy is intended to have universal application it is particularly relevant to mathematics where most significant behaviours appear to have cognitive origins."

In extending the application of taxonomies to student use it must not be assumed that what can be used profitably by teachers is necessarily effective when used by students. Secondly it must not be assumed that teachers and scholars will assess questions in the same way. An attempt should be made to ascertain what influences the student to decide how he should classify a question. Knowledge on these matters can most properly be ascertained from empirical research.

Thus from both the lack of empirical studies in

taxonomic matters on mathematics questions and from the very nature of the enquiry itself, i.e. ascertaining pupil perception of questions, it is essential to undertake an empirical study.

Data required for Investigating the Problem

In order to investigate the aims empirically, the Senior Certificate Mathematics Higher Grade pupils at eight schools were asked to answer a common question paper of a traditional nature set by the present writer. In addition they were asked immediately after the examination to complete a questionnaire in which they were required to classify the questions they had just answered. The teachers who taught Mathematics Higher Grade at these schools at this level were also asked to classify the questions. Permission to use the pupils and schools concerned was obtained from the Education authorities under whose jurisdiction they fell.

The choice of an ex-post facto method of investigation

The ex-post facto research method was chosen because:

1. it yields useful information about the nature of associated factors, and hence it is a valuable exploratory tool giving a fruitful source of hypotheses that can be tested more rigorously later;
2. there is no artificiality in the research proceedings - in the present study the examination was part of the normal examination at the schools chosen; and
3. it is particularly appropriate when simple cause-and-effect relationships are being considered.

(Cohen and Manion, 1980 pp 149 - 150)

However, certain disadvantages have to be taken into account when conclusions are drawn:

1. There is a lack of control on the variables.
2. One cannot be certain whether the causative factor has been included or even identified - there may be a multitude of causative factors.
3. When a relationship has been discovered there is the problem of deciding which is the cause and which is the effect.
4. There is the difficulty of interpretation and the danger of the assumption that because A precedes B then A causes B.

Bearing in mind that this research is exploratory in nature it was considered that the ex-post facto design is a satisfactory one for present purposes.

Chapter 3

The Design of the Study

The Selection and Construction of Instruments for collecting Data

1. Selection and design of the Mathematics Higher Grade examination paper for Standard 10 pupils. (For paper referred to see Appendices A1 and A2). Reasons for the choice of paper and choice of timing are as follows:-
 - a) The present writer was examiner for Mathematics Higher Grade First Paper for the Cape Education Department from 1979 to 1981, and thus had first-hand experience at setting papers which could be written in a number of schools at the Standard Ten level in algebra.
 - b) It was decided to use the mid-year examination because it was felt that most schools would wish to set their own September papers, being the last examination before the external paper at the end of the year.
 - c) The Algebra paper was chosen because most schools finish the work in algebra before completing the Geometry and Trigonometry sections of the course. This was found to be the case - only minor adjustments had to be made to the paper in order to accommodate individual schools.
 - d) The paper set was of the normal format because it is not wise to ask students to answer experimental papers in their Standard Ten year when they require practice in answering the type of paper they will be called upon to answer at the end of the year. In addition proven research on the more usual style of paper may be expected to receive greater acceptance

than on papers of an experimental format.

2. Design of the questionnaires

- a) Methodological Considerations to be taken into account when using a questionnaire in research.

According to Cohen and Mannion (1980 pp 82 -5) the decision of a questionnaire should satisfy the following criteria:

- i) It should be clear, unambiguous and uniformly workable.
- ii) The following kinds of questions should be avoided:
 - leading;
 - highbrow;
 - complex;
 - those that use negatives; and
 - open-ended
- iii) The working should be simple and the overall design easy to follow.
- iv) The questionnaire should look easy.

All these methodological considerations were implemented in the present investigation, although Question C (ii) (see Appendices B3 and B5) could have been condensed. Detailed comment on the questionnaire follows in the next section.

- b) The construction and design of questions in the questionnaire.

The teachers' and pupils' questionnaires with letters of explanation accompanying them are given in the following appendices:

- B1 and B2 : Letter to Teachers and teachers' questionnaire in both English and Afrikaans.
- B3 and B5 : Letter to pupils and pupils' questionnaire in both English and Afrikaans.
- B4 and B6 : Questionnaire reply sheets in English

and Afrikaans.

(i) Use of language

It was realised that the formal language used on p.5 to describe the taxonomic classification would be counter-productive if used in the questionnaire. The words "taxonomic classification" were replaced by the more meaningful "Assessment Symbol", and far simpler descriptions were given to the taxonomic classifications. No pilot study was conducted, but in order to ascertain whether the language was clear enough in the instructions to the pupils (and to the teachers!), they were discussed with the head of the mathematics department at each school. One must remember that pupils who study Mathematics Higher Grade at Senior Certificate Level generally belong to the upper half of the general academic stream in Standard Ten. The opinion on the clarity of the instructions and questions would appear to have been justified as only 5% of the responses were filled in incorrectly. The teacher in charge of each mathematics department agreed to control the filling in of the pupils' questionnaires, and this arrangement seems to have been successful.

(ii) The questions selected for the pupils' questionnaire (Appendices B3 and B5)

"A. Have you seen this type of question before?"

The first essential of any process objective-based taxonomy is to establish whether the pupil has been exposed to the question before. Hence question A.

After a positive response the pupil was asked to reply to question B.

"B(i) Did you learn to answer the question well?"

This question sought to obtain additional information over and above that contained in the taxonomy, but which might help with the interpretation of results. It was afterwards considered that this information was extraneous to the research and had no real affect on the definition "being exposed to the type of question before." Thus the replies to this question were ignored.

"B(ii) Did you find the working-out easy?"

A positive reply to this question resulted in the assessment being that of "Knowledge", while a negative classified the question as being "Comprehension and Application."

Following a negative response to A, the pupils were directed to answer Question C.

"C(i) Was it easy to make the connection between your knowledge and the new situation?"

C(ii) Did you find that you had to analyse the question carefully and that it was only after a lot of thought that you could bring all your knowledge together to answer the question even though you recognised certain aspects that were needed in the solution straightaway."

It can be seen that these questions correspond to the classification Analysis and Synthesis. In these two questions there was an attempt to distinguish between Analysis and Synthesis as the latter is held to be a higher order ability requiring a greater exercise of thought processes than the former in Bloom's original taxonomic

hierarchy. It was found however, that the number of responses to these two questions necessitated their amalgamation in order that meaningful statistical work could be carried out.

"C(iii) Did you find that the question asked something completely different from what you are used to?"

This question clearly reflects the classification Inventiveness.

(iii) The questions selected for the teachers' questionnaire (Appendices B1 and B2)

These are virtually the same as those in the pupils' questionnaire. The only difference being in B(i) and B(ii) where question B(i) refers to the Knowledge classification, and B(ii) to the classification "Comprehension and Application."

3. Organisation of the Questionnaire Reply Sheets (Appendices B4 and B6)

The instructions for completing the reply sheet were stated in both the Teachers' and Pupils' Questionnaires. In instruction 2 in Appendices B1 and B2 on p.80 and p.86 the following statement occurs.

"Hand out the questionnaire reply sheets informing the pupils that the fact that half have reply sheet 2 on top of reply sheet 1 is deliberate."

The reasoning for organising the reply sheets in this way was to allow for the natural tendency to hurry the answers to a questionnaire at the end, especially if answering it proves to be laborious. In the event no teacher reported any difficulty,

except for the case of one pupil who suffered from deafness, but her reply sheet was filled in correctly.

List of the Measures to be obtained using the Measuring Instruments

The following data was sought and obtained:

1. The pupils' scores for the examination papers.
2. The distribution of classifications for each of the 51 questions on the mathematics examination paper by the teachers.
3. The distribution of assessments for each question on the mathematics examination paper by the pupils.

Criteria for Assessing the Important Properties of the Pupils' Performance Scores

1. Sampling Validity

a) Content

A consideration of the table of specifications drawn up by the present writer immediately after the paper had been set shows that all aspects of the syllabus in Algebra had been covered (See Appendix C p.102). It is also apparent that no one process objective is dominant. As would be expected the objective Inventiveness does not have many marks allocated to it, but one cannot ask many questions at this level in an examination. Thus it can be concluded that both the content and the taxonomic levels have been fairly sampled.

b) Population

The population may be regarded as having been fairly sampled because the pupils were drawn from:

- i) two different education departments; and
- ii) both language groupings; and
- iii) both city and country town situations; and
- iv) a full range of mathematical ability as shown

by the results.

2. Reliability

No reliability coefficient has been calculated because of the difficulty of applying adapted Kuder-Richardson and Split-Half techniques to 51 questions of unequal weighting. Despite this the examination may be regarded as reliable because of the following factors:

- a. The test set was three hours long and this is more likely to be reliable than a short test.
- b. The test was given to a population in which there was a great range of talent.

In addition to the above, scorer reliability was good in that all the scripts were marked by the present writer according to the given memorandum (Appendix A3) to which there was strict adherence.

Criteria for Determining the Important Properties of the Data obtained from the Questionnaires

1. Sampling validity

a. Population

As on p. 14

b. Objectivity

It must be realised that every taxonomic classification involves subjective judgement.

In order to minimise subjectivity an attempt was made:

- i) to make the instructions clear, simple and unambiguous; and
- ii) to make the responses required as simple as possible; and
- iii) to cover the full range of opinion that can be expressed.

As far as (i) is concerned it has already been

demonstrated that there was little confusion. As for (ii) the response required, as shown by the reply sheet (Appendices B4 and B6) was a simple Yes or No. In the case of answers to C it was stipulated that the answers had to be of the form Yes/No/No or No/Yes/No or No/No/Yes. As for (iii) it was not felt necessary in any way to add another category of "Other" as all types of question were included.

2. Reliability

No record of subjective judgement can be regarded as being completely reliable. On a test re-test basis there will always be a change of opinion. The present writer's judgement changed in the space of six months on the classification of 15 of the 51 questions (not having looked at his original classifications during that period). Judgements can only be made in the light of the facts available to the person making a judgement at that particular time, and it must be remembered that the present writer was handicapped in that he was not fully aware as to the way in which the subject had been taught at the various schools. Thus with this variation it is important to obtain assessments from a number of teachers. Although each teacher will make his decision in the light of his teaching of the subject matter which will affect the classifying of questions especially in the "seen before" category, there is an overall consensus of what is to be taught in South African schools because of the conservative influence of the traditional style Senior Certificate paper. Hence a collective judgement does have meaning. Perhaps pupils and teachers involved in the investigation should have answered the questionnaire again after an interval in order to measure

the stability of their responses, but against this it must be realised that a separation in time between surveys usually results in added learning having taken place and this could also affect the assessments.

Selection of the Schools and the Population Samples

1. Initial selection

Eight schools were chosen according to the following criteria:

- a. It was thought desirable that the schools should not all fall under the same education authority, and hence both the Cape Education Department and the Department of Internal Affairs were asked permission to approach schools under their authority.
- b. Both official language groupings should be represented amongst the pupils tested.
- c. Both the urban and country town situation was to be represented.

2. Final selection of the schools for data collection purposes.

Each of the heads of the mathematics departments in collaboration with their principals was asked if they would allow the present writer to set and mark their mid-year Algebra examination paper in Standard Ten. Seven of the schools acceded to the request, but at one of these the pupils were able to answer only a third of the paper, with the result that it was afterwards decided that their results and classifications would not be included in the research. The eighth school requested that the paper be used as a pre-test to the examination set in the school. This request was acceded to, but because the candidates wrote the examination

- under different conditions from those of the other schools it was again decided that their results would not be used.

3. Pupil groupings within the six schools

In all, 259 candidates in the remaining six schools set the examination and gave their assessment of each question. Of these, the results of one pupil's examination and his classification of the questions was lost. Apposite background information on the schools and pupils is given in Table 3.1.

School	No. of groups	No. of pupils	No. of teachers
A	2	39	2
B	2	39	2
C	2	37	1
D	1	19	1
E	4	E ₁ - 17	4
		E ₂ - 25	
		E ₃ - 23	
		E ₄ - 30	
F	1	29	1
Totals	12	258	11

Table 3.1 Background information on Mathematics Higher Grade groupings and teachers at the schools at which research was carried out

Comments on the groupings

School A : The basis of the grouping is not known to the present writer, nor which pupils belong to which group or taught by which teacher. The results were taken as one group. A third teacher gave his taxonomic classification of the questions.

School B : The grouping is based on known ability at the end of Standard Seven. It was not known

which pupils belonged to which groups taught by which teacher. The results were taken as one group.

School C : Another experienced teacher also assessed the questions. It is not known which teachers' classification was made by the teacher who taught the groups. Results were taken as one group.

School E : Groups were divided according to known ability; pupils in group E_1 having the highest ability in Mathematics. The teachers of groups E_1 , E_2 and E_3 gave their assessments.

School F : The teacher at this school had taught the pupils for only two months prior to the examination and his assessments were based on the students' responses in revision.

Modifications made to the Examination Paper

1. In discussions with the teacher in charge of Mathematics at each school it was ascertained that schools C, D and F had completed the entire syllabus in Algebra, while the other schools needed only to complete the section on Mathematical Induction. It was agreed that the pupils at these schools would be given a second question on quadratics in place of that on Mathematical Induction. Hence the choice between the two Question 9's (see Appendices A1 and A2).
2. The question paper was perused by all the mathematics teachers involved with the experiment and was approved by all of them. Class-group D, however, were told not to attempt Question 3 (c) and were required to answer Questions 9a and 9b(ii) as well as the question on Mathematical Induction.
3. Subsequent to the examination being written the

present writer was informed that groups E_3 and E_4 were not able to answer Questions 5(c) to 6(c) on Indices and Logarithms, and had been given other questions in their place.

It was felt that the effect of the changes in 2 and 3 above were minor, and so results of these three groups were included in the research.

Selection of the Statistical Treatment of the Results

1. Frequencies were subjected to χ^2 tests unless the number of frequencies was such that F_t was less than 5. Under these circumstances Fischer's Exact Probability test was used.
2. Differences in mean scores between questions classified at the various taxonomic levels were subjected to t-tests. Here it was assumed that the data was homoscedastic since distribution of pupils' scores are usually normally distributed.

List of Hypotheses

The following hypotheses were tested using the data collected

1. Hypotheses concerned with significant agreements on the taxonomic classification of questions by pupils and teachers.
 - H1. That when mathematics teachers are asked to classify individual examination questions as one of K, CA, AS or I, they will reach a significant level of agreement on the taxonomy of the questions ie there will be significantly more definite as opposed to split resultant classifications compared with a random distri-

bution.

- H2. That mathematics teachers will classify questions as K with a significantly greater degree of unanimity than questions involving higher order process objectives.
 - H3. That there will be significantly more definite as opposed to split resultant classifications on the 51 mathematics examination questions compared with a random distribution.
 - H4. That pupils will classify questions as K with a significantly greater degree of agreement than questions involving higher order process objectives.
 - H5. That mathematics teachers will be able to classify examination questions with a significantly greater degree of agreement than their pupils.
 - H6. That there will be a significant measure of agreement between teachers and their pupils on the classification of K/CA questions (questions exposed to before) as opposed to AS/I questions (questions not exposed to before).
 - H7. That there will be a significant (non-random) measure of agreement between teachers and their pupils on the classification of questions as K rather than as CA.
 - H8. That there will be a significant (non-random) measure of agreement between teachers and their pupils on the classification of questions as AS rather than as I.
2. Hypotheses concerned with Significant Agreements on the Resultant Taxonomic Classification of Questions when the Pupils are divided into Quintiles according to Performance Scores.
- H9. That when pupils are separated into quintiles according to their mathematical performances

on an examination paper, a significant difference will occur between the number of definite resultant classifications of questions by pupils and the number of split resultant classifications in each quintile compared with a random distribution for those quintiles.

- H10. That there is a significant non-random difference between the number of definite resultant classifications and the number of split resultant classifications between pupils of higher and lower ability divided according to performance score quintiles.
- H11. That there is a significant difference between the number of definite resultant classifications and the number of split resultant classifications by the pupils at each quintile level compared with those of the teachers resultant classifications.
- H12. That there is a significant difference amongst the quintiles between the number of resultant classifications given by pupils as being "exposed to before" (K/CA) and those "not exposed to before" (AS/I).
- H13. That there is a significant difference between the number of resultant classifications given by pupils as "exposed to before" (K/CA) as opposed to those given as "not exposed to before" (AS/I) in each quintile and the resultant classifications given by the teachers.
- H14. That there is a significant difference between the number of resultant classifications given as K and those given as CA between the pupil quintiles.
- H15. That there is a significant difference between the distribution of the teachers' resultant

classification of questions as K as opposed to CA and each pupil quintile's classification distribution of such questions.

H16. That there is a significant difference between the number of resultant classifications given as AS and those given as I between the pupil quintiles.

H17. That there is a significant difference between the teachers' resultant classification of questions as AS as opposed to I and each pupil quintile's classification of such questions.

H18. That there is a significant difference between the quintiles as far as agreement with the resultant classifications of the teachers is concerned.

3. Hypotheses concerned with the Significant Differences in the Actual Taxonomic Classifications of Individual Questions between Pupils divided according to Quintile and the Relationships of these Differences to Performance Scores

H19. That in each question there is a significant difference between the classifications given by the pupils in the different quintiles.

H20. That there is a significant difference in the average percentage performance score between quintiles in which there is a significant classification difference and quintiles where there is no significant classification difference.

H21. That there is a significant difference in average percentage score for each quintile - pairing between those difference scores where the classification difference is regarded as significant, and the difference scores where the classification is regarded as not significant.

H22. That in each question, as ability measured by

performance scores decreases, the number of classifications given as AS and K each decrease significantly and those given as CA and I each increase significantly.

4. Hypotheses concerned with the Averages of the Performance Scores at the Different Taxonomic Levels

H23. That when the averages of the questions classified at the different taxonomic levels by the teachers are considered there is a significant difference between the averages of those questions classified as K, CA and AS/I.

H24. That when the averages of the questions classified at the different taxonomic levels by the pupils are considered, there is a significant difference between the averages of those questions classified as K, CA and AS/I.

H25. That in each question there is a significant difference in the average performance score of the pupils who classify the questions at the different taxonomic levels.

H26. That when the averages of the questions classified at the different taxonomic levels by the teachers are considered by quintile there is a significant difference between the averages of those questions classified as K, CA and AS/I in each quintile.

Chapter 4

Results : Organisation of the Data Obtained The Pupils' Examination Data

After the scripts had been marked, the scores for each question, together with any errors made in that question, were recorded. The errors were analysed for each school and the information obtained therefrom was sent to the relevant school.

The scores were then arranged in order of merit and divided into quintiles - the ranges and means of which are given in Table 4.1

Quintile	Lowest % Mark	Highest % Mark	Mean
Q1	57	92	69,4
Q2	49,5	57	52,5
Q3	34	49,5	39,6
Q4	23,5	34	28,8
Q5	4	23,5	17,3
Total	4	92	41,1

Table 4.1 Basic statistical data of the performance scores of each quintile grouping.

The number of pupils in each quintile is given in Appendix D1, the variation being due to the factors mentioned on pp19-20. The pupils' classifications were then collated with their scores and recorded for each class group in each quintile. Thus the data was readily available in sub-divided form either by class group or by quintile.

The Data from the Teachers' Questionnaire

The data on teachers' classifications were collated question by question and the resulting distributions are given in Appendix D2 on p104.

Teachers should have a fairly clear idea on how to classify questions, but even with this there is certainly no unanimity of assessment - this being achieved on two questions only. This tends to confirm the finding of Willmot and Hall (1973 p.117) that there is much lack of agreement on the classification of questions into various taxonomic levels. They state further that there is greater consensus of opinion on the knowledge category. This is also confirmed when it is noted that of the 14 teachers' classifications, ten or more appear in the Knowledge category 11 times, and in all the other higher process objectives twice only.

Because of this lack of agreement some decision has to be taken as to what constitutes a final classification, to which the present writer has given the term resultant classification. The following conclusions were reached:

1. For a resultant classification to be regarded as definite it must have received more than 50% of the assessments given by the teachers.
2. If no process objective received more than 50% of the assessments then the resultant classification would be regarded as a split classification in favour of whichever process objective received the greatest number of assessments.
3. If equal numbers of assessments were given to two particular process objectives then this would be indicated as say CA/AS as in Question 2a(iv).

The Data from the Pupils' Questionnaire

The raw data was organised in the same way as that from the Teachers' Questionnaire.

Considering the raw data on pupils' classifications given in Appendix D3 on p106, it is immediately evident that there was no unanimity on the pupils' classification of questions according to process objectives. Indeed it is only on Question 2b(v) that more than 80% of the pupils classified the question as belonging to one particular category. Pupils' classifications were also considered for each quintile grouping (See Appendix D4).

Because of this variation use was again made of resultant classifications which were defined in the same way as teachers' resultant classifications.

Agreement between Teachers' and Pupils' Resultant Classifications

Judging from the resultant classifications it would seem that there are three states of agreement:

1. Definite Agreement where exactly the same definite resultant is given by both teachers and pupils.
2. A Measure of Agreement where there is some connection between the pupils' and teachers' resultant classifications.
 e.g. a. Split K and K
 b. Split CA and Split CA
 c. Split AS and AS/CA
3. No Agreement at all - completely different resultants

In this research the first two categories were combined so that agreement of some nature between teachers' and pupils' classifications was compared with disagreement.

Questions Chosen for Detailed Analysis

In order to analyse the classifications made by the pupils on individual questions in and between each quintile it was decided to consider the pupil classifications of a large sample of the questions, selected on the bases given below.

1. Selection of questions to be analysed

In selecting the questions it was decided to analyse those where the distribution of classifications was such that a χ^2 test could be used for a minimum of three sub-divisions of process objectives, where the frequencies in observed cells was always 5 or more. This meant that 23 questions would be analysed. On further investigation it was realised that only one question in which the candidates scored an average of more than 60% was included amongst these questions, and it was decided to include two more in this bracket.

A list of questions used appears in Table 4.2 along with relevant information on average % score, and on content and process objective resultant classifications.

From the table it can be seen that a full range of questions from easy to difficult was considered with the lowest average being 5% and the highest 85%. With the mean of the entire examination being 41.1% it is not surprising that 14 of the 25 questions chosen have an average performance score of 41% or less. The questions chosen thus represent a fair sampling of easy to difficult questions.

On content the only aspect not sampled was that of Mathematical Induction. The reason for this was that only 100 pupils answered the question on this

aspect, and when these are divided into five quintiles which are each classified four ways it is extremely unlikely that all theoretical frequencies in a χ^2 test will be greater than 5.

The process objectives are fairly sampled. The teachers classifications are approximately evenly divided amongst K, CA and AS while both questions which were classified as I by the teachers are analysed. The pupils' classifications reflect the dominance of the Knowledge and Inventiveness categories although the classification CA is also sampled. The complete lack of questions classified as AS in the sample is not surprising considering that only one out of all the questions has a resultant pupil classification of AS.

Thus the 25 questions chosen may be regarded as being an adequate sample of the questions reflecting the examination paper as a whole.

2. Data Considered and Statistical Tests Used in the Analysis

In Appendix D5 the following data is given for each of the 25 questions chosen:

- a. The teachers' and pupils' resultant classifications.
- b. The classifications by each candidate totalled by quintile.
- c. The average % score for each quintile.

The following statistical tests were used:

- a. A χ^2 test on all the data to see whether there is an overall significant difference in question classification between pupils of high ability compared to those of lower ability.

Overall Classification of Questions by Objectives

<u>Questions</u>	<u>Average %</u>	<u>Content</u>	<u>Teachers' Resultant Process</u>	<u>Pupils' Resultant Process</u>
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Questions where use could be made of all four process objective classifications

1a(i)	53	Relations and Functions	K	K
(ii)	56		K	Split K
(iii)	58		K	Split K
1c	73		CA	Split K
2a(iv)	58	Indices and Logs	CA/AS	Split I
b(ii)	45		CA	Split K
8(ii)	16		AS	I
9b(ii)	23		CA	Split I
1d	48	Equations and Inequalities	Split AS	Split I
4b	31		CA	Split CA
4c(i)	26		AS	Split CA
7a(ii)	17		Split AS	Split K
7c(i)	27	Sequences and Series	CA	Split CA

Questions where use had to be made of three process objective classification sub-divisions

a) K, CA, AS/I

2a(iii)	85	Relations and Functions	K	K
2b(i)	71		K	K
1e	55	Indices and	K	K
6a	29		CA	Split CA
6b	55	Logarithms	K	K
3b	32		CA	Split K
7a(ii)	17	Equations and Inequalities	Split AS	Split K
7b	40		K	Split K

b) K/CA, AS, I

2b(vi)	10	Relations and	Split CA	Split I
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Overall Classification of Questions by Objective

<u>Questions</u>	<u>Average %</u>	<u>Content</u>	<u>Teachers' Resultant Process</u>	<u>Pupils' Resultant Process</u>
8(i)	41	Functions	Split AS	Split I
3c	13	Equations and	I	I
4c(ii)	5	Inequalities	I	I

Table 4.2 Questions chosen for analysis : Average percentage scores and Classifications of each according to content and process objectives

- b. Chi-squared tests in order to ascertain significant differences between the classifications between pairs of quintiles. All ten pairing of quintiles were considered for each question.

The significance of the results of these tests as well as the difference in average % score between each quintile are also given in Appendix D5.

Average Performance Scores

As well as comparing Average Performance Scores between questions which have differing Resultant Classifications it was also decided to compare the Pupils' Performance Score based on their own classifications for each question. The data for this is given in Appendix D6.

In order to determine whether there is any significant trend between the Average Performance Scores of pupils with higher ability compared with those of lower ability, when the averages for different resultant classifications are compared, the averages for each quintile were calculated and are given in Appendix D7.

Chapter 5

Results : Confirmation or Refutation of Hypotheses

Hypotheses Concerned with Significant Differences and Agreements on the Taxonomic Classification of Questions by Pupils and Teachers

Data used to test the Hypotheses in this section is given in Appendices D2 and D3.

- H1. That when mathematics teachers are asked to classify individual examination questions as one of K, CA, AS or I, they will reach a significant level of agreement on the taxonomy of the questions ie there will be significantly more definite as opposed to split resultant classifications compared with a random distribution.

	Definite Resultants	Split Resultants
Observed	37	14
Random	26*	25*

* Theoretically $25\frac{1}{2}$, but rounded to the nearest whole number in such a way as to make the χ^2 test a little more stringent, i.e. 26-25, rather than 25-26.

Table 5.1 Distribution of teachers' definite and split resultant classifications compared with a random distribution

Using a χ^2 test the difference is significant ($P=0.05$)
The hypothesis is confirmed.

- H2. That mathematics teachers will classify questions as K with a significantly greater degree of unanimity than questions involving higher order process objectives.

	Definite Resultants	Split Resultants
Knowledge	23	1
Higher Order Process Objectives	14	13

Table 5.2 Distribution of teachers' definite and split resultant classifications according to process objectives comparing Knowledge with Higher Order.

Using a χ^2 test the difference is significant ($P=0,005$)
The hypothesis is confirmed.

- H3. That there will significantly more definite as opposed to split resultant classifications on the 51 mathematics examination questions compared with a random distribution.

	Definite Resultants	Split Resultants
Observed	22	29
Random	25	26

Table 5.3 Distribution of pupils' definite and split resultant classifications compared with a random distribution.

Using a χ^2 test there is no significant difference.
The hypothesis is refuted.

- H4. That pupils will classify questions as K with a significantly greater degree of agreement than questions involving higher order process objectives.

	Definite Resultants	Split Resultants
Knowledge	16	16
Higher Order Process Objectives	6	13

Table 5.4 Distribution of pupils' definite and split resultant classifications according to process objectives comparing Knowledge with Higher Order.

Using a χ^2 test the difference is not significant.
The hypothesis is refuted

- H5. That mathematics teachers will be able to classify examination questions with a significantly greater degree of agreement than their pupils.

	Definite Resultants	Split Resultants
Teachers' Classifications	37	14
Pupils' Classifications	22	29

Table 5.5 Distribution of teachers' and pupils' definite and split resultant classifications

Using a χ^2 test the difference is significant ($P=0,005$)
The hypothesis is confirmed

- H6. That there will be a significant measure of agreement between teachers and their pupils on the classification of K/CA questions (questions exposed to before) as opposed to AS/I questions (questions not exposed to before).

	<u>Teacher and Pupil Classifications</u>	
	<u>Agreement between</u>	<u>Disagreement between</u>
Observed	44	7
Random	26	25

Table 5.6 Distribution of agreement in resultant classifications of questions between teachers and pupils on K/CA as opposed to AS/I classifications

Using a χ^2 test the difference is significant ($P=0,001$)
The hypothesis is confirmed

- H7. That there will be a significant (non-random) measure of agreement between teachers and their pupils on the classification of questions as K rather than as CA.

	<u>Teacher and Pupil Classifications</u>	
	<u>Agreement between</u>	<u>Disagreement between</u>
Observed	31	10
Random	21	20

Table 5.7 Distribution of agreement in resultant classifications of questions between teachers and pupils on K as opposed to CA classifications

Using a χ^2 test the difference is significant ($P=0,05$)

The hypothesis is confirmed

- H8. That there will be a significant (non-random) measure of agreement between teachers and their pupils on the classification of questions as AS rather than as I.

	Agreement	Disagreement
Observed	2	10
Random	6	6

Table 5.8 Distribution of resultant classification agreements between teachers and pupils on questions classified as AS or I.

Using a Fischer's Exact Probability Test the difference is not significant.

The hypothesis is refuted.

Hypotheses concerned with Significant Agreements on the Resultant Taxonomic Classification of Questions when the Pupils are divided into Quintiles according to Performance Scores.

Raw data is given in Appendix D4

- H9. That when pupils are separated into quintiles according to their mathematical performances on an examination paper, a significant difference will occur between the number of definite resultant classifications of questions

<u>Group/Quintile</u>	<u>Definite Resultants</u>	<u>Split Resultants</u>
Q1	34	17
Q2	30	21
Q3	25	26
Q4	23	28
Q5	19	32
Teachers	37	14
Random	26	25

Table 5.9 Distribution of Definite and split resultant classifications according to each pupil quintile grouping and according to the teachers, each compared with a random distribution.

by pupils and the number of split resultant classifications in each quintile grouping compared with a random distribution for those quintiles.

The data relevant to this hypothesis is summarised in Table 5.9.

Using χ^2 tests for each quintile distribution there is no significant difference.

The hypothesis is refuted.

H10. That there is a significant non-random difference between the number of definite resultant classifications and the number of split resultant classifications between pupils of higher and lower ability divided according to performance score quintiles. Data is given in table 5.9.

Using χ^2 tests:

- a. the overall difference in the results is significant ($P = 0,01$);
- b. the difference in results between (i) Q1 and Q4 is significant ($P = 0,05$);
(ii) Q1 and Q5 is significant ($P = 0,01$);
(iii) Q2 and Q5 is significant ($P = 0,05$).

The hypothesis is confirmed for quintile groups

whose performance scores are markedly different.

H11. That there is a significant difference between the number of definite resultant classifications and the number of split resultant classifications by the pupils at each quintile level compared with those of the teachers resultant classifications. The data is given in table 5.9.

Using χ^2 tests:

- a. Q1 and Q2 are not significantly different from the teachers' classifications;
- b. the following are significantly different from the teachers' classifications
 - Q3 ($P = 0,05$)
 - Q4 ($P = 0,01$)
 - Q5 ($P = 0,001$)

The hypothesis is confirmed for pupils of lower ability.

<u>Quintile</u>	<u>Classifications</u>			
	<u>Exposed to before</u>	<u>Not exposed to before</u>		
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>
Teachers	24	16½	8½	2
Q1	39	3	6	3
Q2	33	9	2	7
Q3	25½	14	1	10½
Q4	16	21½	1	12½
Q5	12½	17	0	21½

Table 5.10 Distribution of resultant classifications of questions by pupil quintile.

Note: When using χ^2 on these data the classifications which are not whole numbers were changed to the nearest whole number in accordance with the principle of not affecting marginally significant results positively.

H12. That there is a significant difference amongst the quintiles between the number of resultant

classifications given by pupils as being "exposed to before" (K/CA) and those "not exposed to before" (AS/I).

The data is given in table 5.10.

Using χ^2 tests:

- a. There is no significant difference from Q1 through to Q4.
- b. Q5 classifications are significantly different from:
 - Q1 ($P = 0,025$) and
 - Q2 ($P = 0,025$)

The hypothesis is confirmed only for the pupils in the lowest ability range according to performances scores compared with the upper 40% of all the pupils.

H13. That there is a significant difference between the number of resultant classifications given by pupils as "exposed to before" (K/CA) as opposed to those given as "not exposed to before" (AS/I) in each quintile and the resultant classifications given by the teachers.

The data is given in table 5.10.

Using χ^2 tests there are no significant differences.

The hypothesis is refuted.

H14. That there is a significant difference between the number of resultant classifications given as K and those given as CA between the pupil quintiles.

The data is given in table 5.10.

Using χ^2 tests:

- a. The overall difference is significant ($P = 0,001$)
- b. Q1 distribution is significantly different from Q3 ($P = 0,005$)
 - Q4 ($P = 0,001$)
 - Q5 ($P = 0,001$)

- c. Q2 is significantly different from Q4 ($P=0,005$)
Q5 ($P=0,005$)

The hypothesis is confirmed overall and specifically for five of the possible ten comparisons that can be made between quintile groups, these pairing differences coming from non-consecutive quintiles.

H15. That there is a significant difference between the distribution of the teachers' resultant classification of questions as K as opposed to CA and each pupil quintile's classification distribution of such questions.

The data is given in table 5.10.

Using χ^2 tests:

- a. The difference in distribution between the teachers' classifications and that of Q1 is significant ($P = 0,001$)
- b. There are no other significant differences.

The hypothesis is confirmed only for pupils in Q1.

H16. That there is a significant difference between the number of resultant classifications given as AS and those given as I between the pupil quintiles.

The data is given in table 5.10.

Using Fischer's Exact Probability test:

- a. Q1 is significantly different from Q3 ($P = 0,02$)
Q4 ($P = 0,01$)
Q5 ($P = 0,001$)
- b. No other significant differences occur.

The hypothesis is confirmed for the distribution of Q1 classifications compared with the classifications of the lowest three quintiles divided according to performance scores.

H17. That there is a significant difference between the teachers' resultant classification of questions as AS as opposed to I and each pupil quintile's classification of such questions.

The data is given in table 5.10.

Using Fischer's Exact Probability test:

- a. The distribution of the teachers' classifications is significantly different from:
 - Q2 ($P = 0,02$)
 - Q3 ($P = 0,002$)
 - Q4 and Q5 ($P = 0,001$)
- b. The distribution of teachers' classifications is not significantly different from Q1.

The hypothesis is confirmed for all but the pupils with the highest performance scores.

H18. That there is a significant difference between the quintiles as far as agreement with the resultant classifications of the teachers is concerned.

<u>Grouping</u>	<u>Agreement</u>	<u>Disagreement</u>
Q1	32	19
Q2	35	16
Q3	28	23
Q4	24	27
Q5	19	32

Table 5.11 Distribution of agreement and disagreement on resultant classifications between teachers and the different quintile groups

Using χ^2 tests:

- a. The overall difference is significant ($P = 0,05$)
- b. Q5 distribution is significantly different from Q1 ($P = 0,001$)

The hypothesis is confirmed overall, and specifically for students with the lowest scores compared with the classifications of the upper 40% of the pupils.

Hypotheses Concerned with the Significant Differences in the Actual Taxonomic Classifications of Individual Questions between Pupils divided according to Quintiles

and the Relationship of these Differences to Performance Scores.

The new data for the hypotheses in this section is given in Appendix D5.

H19. That in each question there is a significant difference between the classifications given by the pupils in the different quintiles.

<u>Overall Difference</u>	<u>Number of Questions</u>
Significant : P = 0,001	16
P = 0,005	1
P = 0,025	3
P = 0,05	<u>2</u>
Sub-total	22
Not significant	<u>3</u>
Total	25

Table 5.12 Results of χ^2 -tests on the pupil classification data given in each question in Appendix D5.

Using χ^2 -tests the overall differences in classifications is significant in 22 out of the 25 questions.

The hypothesis is confirmed.

H20. That there is a significant difference in the average percentage performance score between quintiles in which there is a significant classification difference and quintiles where there is no significant classification difference.

Using a random sample t-test the overall difference in means of average % difference scores between significantly and non-significantly different classification distributions is significant ($P = 0,01$).

The hypothesis is confirmed.

H21. That there is a significant difference in

<u>Classifications between Quintiles</u>					
<u>Significantly different</u>			<u>Not significantly different</u>		
<u>Questions</u>	<u>N</u>	<u>Mean of Average % Difference Scores</u>	<u>N</u>	<u>Mean of Average % Difference Scores</u>	
1a(i)	4	31,5	6	14,0	
(ii)	5	39,0	5	14,8	
(iii)	6	37,8	4	10,3	
c	2	38,0	8	15,0	
d	4	21,0	6	11,3	
2a(iv)	6	37,8	4	10,3	
b(ii)	4	36,5	6	15,0	
4b	5	33,8	5	17,8	
c(i)	3	18,0	7	10,8	
7a(ii)	2	59,0	8	18,8	
c(i)	2	39,0	8	18,8	
8(ii)	7	30,6	3	5,0	
9b(ii)	5	42,0	5	17,2	
1e	8	45,6	2	17,0	
2a(iii)	6	26,3	4	4,5	
2b(i)	2	35,5	8	17,5	
3b	5	23,4	5	16,2	
6a	0	-	10	28,0	
b	6	48,8	4	14,3	
7b	7	33,9	3	10,8	
c(ii)	5	38,0	5	15,2	
2b(vi)	4	23,0	6	6,0	
3c	5	25,4	5	15,8	
4c(ii)	2	1,0	8	9,5	
8(i)	5	48,0	5	16,8	

Table 5.13 Differences in averages by question between significantly and non-significantly different pupil question classifications between quintiles on the 25 questions as selected

average percentage score for each quintile-pairing between those difference scores where the classification difference is regarded as significant and the difference scores where the classification is regarded as not significant.

The mean of the average difference scores for the significantly different classifications between quintiles is always higher than for the non-significantly different classifications. This means that this result is definitely highly significant,

<u>Quintiles</u>	<u>Classification between Quintiles</u>			
	<u>Significantly different</u>		<u>Not significantly different</u>	
	<u>N</u>	<u>Mean of Average % Difference Scores</u>	<u>N</u>	<u>Mean of Average % Difference Scores</u>
Q1 - Q2	4	32,8	21	16,2
Q2 - Q3	14	35,4	11	20,4
Q1 - Q4	18	42,3	7	31,0
Q1 - Q5	22	49,8	3	39,7
Q2 - Q3	3	18,0	22	9,8
Q2 - Q4	13	23,4	12	17,3
Q2 - Q5	17	36,0	8	18,0
Q3 - Q4	6	13,8	19	9,8
Q3 - Q5	9	28,6	16	15,1
Q4 - Q5	4	12,8	21	9,3
	$\Sigma N=110$	Mean=34,9	$\Sigma N=140$	Mean =14,9

Table 5.14 Differences in averages between quintiles on 25 questions as selected.

especially considering the large difference in the means of these average % difference scores.

The hypothesis is confirmed.

H22. That in each question, as ability measured by performance scores decreases, the number of classifications given as AS and K each decrease significantly and those given as CA and I each increase significantly.

While doing the χ^2 tests for H20 and H21 it was noted that a significant change in classification between quintiles occurred only when there was a change of at least 20% in the number of classifications in any one category. All changes of this significance were recorded and are given in table 5.15.

<u>Classification</u>	<u>Classification change between quintiles as ability decreases</u>		
	<u>Significant increase</u>	<u>Significant decrease</u>	<u>Total</u>
K	0	78	78
CA	29	2	31
AS	1	19	20
I	50	0	50
K/CA	0	3	3
<u>AS/I</u>	<u>18</u>	<u>0</u>	<u>18</u>
Totals	98	102	200

Table 5.15 Significant changes in pupil classification between quintiles as ability measured by performance score decreases.

Using χ^2 tests comparing each classification with a random distribution.

a. K and AS show significant decreases ($P = 0,001$)

b. CA and I show significant increases ($P = 0,001$)

The hypothesis is confirmed.

Hypotheses concerned with the Averages of the Performance Scores at the Different Taxonomic Levels

H23. That when the averages of the questions classified at the different taxonomic levels by the teachers are considered there is a significant difference between the averages of those questions classified as K, CA and AS/I.

<u>Classification</u>	<u>K*</u>	<u>CA*</u>	<u>AS/I</u>
N(Total 51)	25	18	12
Mean	57,2%	34,3%	22,8%

Note * Classifications such as CA/AS had their averages included under both categories.

Table 5.16 Question average scores arranged according to teacher resultant classifications

Using random sample t-tests the difference between the average scores for:

- a. K as opposed to CA is significant ($P = 0,01$)
- b. K as opposed to AS/I is significant ($P = 0,01$)
- c. CA as opposed to AS/I is significant ($P = 0,05$)

The hypothesis is confirmed.

H24. That when the averages of the questions classified at the different taxonomic levels by the pupils are considered, there is a significant difference between the averages of those questions classified as K, CA and AS/I.

Classification	<u>K</u>	<u>CA</u>	<u>AS/I</u>
N (Total 51)	32	9	10
Mean	54,6	27,1	20,5

Table 5.17 Question average scores arranged according to pupil resultant classifications.

Using random sample t-tests the difference between the average scores for:

- a. K as opposed to CA is significant ($P = 0,01$)
- b. K as opposed to AS/I is significant ($P = 0,01$)
- c. CA as opposed to AS/I is not significant.

The hypothesis is confirmed for questions classified as Knowledge when compared with questions given a higher order classification.

H25. That in each question there is a significant difference in the average performance score of the pupils who classify the questions at the different taxonomic levels

Raw data is given in Appendix D6.

Classification	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>
Distribution of Pupil Classifications	4630	3022	1463	2058
Means	69,7	32,4	36,7	15,4

Table 5.18 Distribution of pupil classifications of questions and their averages.

Using random sample t-tests:

a. Differences in means were significant between:

K and CA ($P = 0,01$)

K and AS ($P = 0,01$)

K and I ($P = 0,01$)

CA and I ($P = 0,01$)

AS and I ($P = 0,01$)

b. Difference in means is not significant between
CA and AS

The hypothesis is confirmed for all but one of
the differences.

H26. That when the averages of the questions
classified at the different taxonomic levels
by the teachers are considered by quintile
there is a significant difference between
the averages of those questions classified
as K, CA and AS/I in each quintile.

Raw data is given in Appendix D7.

	<u>Q1</u>	<u>Q2</u>	<u>Q3</u>	<u>Q4</u>	<u>Q5</u>
Average % Scores					
Resultant					
Classification: K	82,3	69,4	54,4	42,6	30,2
CA	63,1	41,4	31,7	20,1	14,9
AS/I	45,5	27,7	17,2	15,3	10,2

Table 5.19 Average question scores by quintile
according to teacher resultant classifica-
tions.

Using random sample t-tests:

a. The difference in average % scores was signifi-
cant for all quintiles between the classifications:

K and CA ($P = 0,01$)

K and AS/I ($P = 0,01$)

b. The difference in average % scores between the
classifications CA and AS/I were

i) significant for Q1 ($P = 0,02$) and Q2 ($P = 0,05$)

ii) not significant for Q3, Q4 and Q5.

The hypothesis is confirmed except for Q3, Q4 and Q5 when comparing the classifications CA and AS/I.

Chapter 6

Discussion and Implications of the Results

Hypotheses concerned with Significant Differences and Agreements on the Taxonomic Classifications of Questions by Pupils and Teachers

1. Summary of Findings

- a. Teacher classifications of mathematics questions show a significant degree of agreement with each other - this being especially true with the questions classified as Knowledge - whereas pupil classifications do not show a significant degree of agreement with each other.
- b. There is significant agreement between teachers and pupils on those questions to which the pupils have been exposed before and, within this category, as to which questions should be classified as K and which CA.
- c. There is a lack of agreement between teachers and pupils as to which questions should be classified as AS and which as I.

2. Interpretation of Findings

The finding that teacher classifications show a significant degree of agreement with each other lends support to the idea that there is a common view towards question-type in Senior Certificate Mathematics papers in South Africa. This common view will, no doubt, be more apparent at the end of the course after there has been much revision and working through questions in past papers, than in mid-year when this research was carried out. Even so, the tendency for South African mathematics teachers to teach to a common pattern

is apparent. The finding that agreement is even more pronounced on classification classified as Knowledge confirms Willmot and Hall's (1973) assertion.

The greater diversity of opinion on questions classified in the higher order categories would seem to reflect the individual approach that each teacher has.

The lack of significant agreement in pupil classifications could be a result of:

- a. a lack of training in how to recognise process objectives; or
- b. a greater heterogeneity of teaching style than that indicated by the answers to the teachers questionnaires - to test this one would have to consider the classifications within individual classes; or
- c. a third factor such as pupil's own ability in the subject causing an interference in the data.

As a teacher it is of great comfort to the present writer to know that the pupils agree with the teachers on the classification of questions to which they have been exposed before.

Regarding the lack of agreement between teachers and pupils on questions classified as AS and I, it would seem as if pupils do not have the depth and breadth of knowledge required to distinguish between questions completely different from what they are used to, and those which they have not seen but where there is an easy connection between what is known and not known. Hence there is a tendency by the pupils to transfer all questions which are not known to the highest of the process

objective classifications.

Hypotheses concerned with Significant Agreements on the Resultant Taxonomic Classification of Questions when the Pupils are divided into Quintiles according to Performance Scores.

1. Summary of findings

- a. Pupils of higher ability agree more in their judgments on the classification of a particular question than pupils of lower ability. This agreement in judgment accords with that of the teachers.
- b. Even in the lowest quintile group where the average percentage score was 17% there is no significant difference between pupil and teacher classifications in respect of whether they had been taught a particular question type or not. This lowest group did, however, differ from the upper two quintile groups in such classification of questions.
- c. As ability decreases, so do the number of resultant classifications given as K, while CA and I classifications increase.
- d. Only the highest ability students give questions a resultant classification of AS.
- e. Pupils of higher ability agree more often with the teachers' classifications of questions than do pupils of lower ability.

2. Interpretation of findings

It would appear that the judgments of pupils of higher ability agree more often with the teachers because they probably have a deeper and broader understanding of the subject matter than those of lower ability. It would seem that they are the only pupils who have enough understanding to be

- able to judge whether a question should be classified in the category AS, whereas pupils of lower ability tend to place all questions which are unfamiliar to them in the category of being completely different from what they are used to - they do not see how knowledge can be transferred.

It would seem, however, that, as far as judging the quality of algorithmic work is concerned, the teacher appears to be thinking of the average pupil whereas the bright pupil regards all algorithmic work as easy, and therefore tends to classify most questions that he has been exposed to before as Knowledge. On the other hand the lower scoring pupils who have greater difficulty with algorithmic processes tend to classify the questions at a higher level. Thus in the "exposed to before" categories the teacher and the brightest pupils do not agree in their classifications of questions.

Hypotheses Concerned with the Significant Differences in the Actual Taxonomic Classifications of Individual Questions between Pupils divided according to Quintiles and the Relationship of these Differences to Performance Scores

1. Summary of findings

- a. It can be seen readily that pupils of higher ability compared with those of lower ability classify questions:
 - i) differently according to process objectives;
 - ii) more often as Knowledge and Analysis and Synthesis; and
 - iii) less often as Comprehension and Application and Inventiveness.

These differences are reflected in the greater

difference in average percentage between quintiles where the classification is significantly different when individual questions are considered separately.

- b. The second conclusion is that there is a relationship between performance scores and classifications made by the pupils.

2. Interpretation of findings

The first conclusion shows that what has already been noted under the change in resultant classification as ability decreases also applies to individual classifications in questions, so that the overall classification of questions is a reflection of the assessment pattern within each question.

The second conclusion raises the important issue of whether the success with which a question is answered will have any effect on the way in which it is classified. It must be remembered that in H21 the influence of the ability of the student as a whole has been removed in that the average scores of significantly different classifications are compared with these of non-significantly different classifications between the same quintiles considered for 25 questions.

Hence the only two variables under consideration are those of the performance scores for particular questions and the classifications for those questions. When the classification variable undergoes a significant change the difference in performance score is significantly higher than when there is no significant change in classification.

Which is the dependent and which the independent

variable cannot be decisively answered by this study. But a tentative conclusion is that it would seem that the difference in classification may have been influenced by the pupils' assessments of their answers as they were allowed to consult their scripts as well as the examination paper when completing the questionnaire reply sheet. A definite answer to this question will have to be found by a more structured study.

Hypotheses concerned with the Averages of the Performance Scores at the Different Taxonomic Levels

1. Summary of findings

- a. The performance scores decrease significantly as the taxonomic resultant classifications increase. The decrease is significant except between CA and AS/I in the lowest three pupil quintiles.
- b. When the averages of the actual pupil classifications were considered this pattern was repeated except that the average for the AS category was slightly higher than that for the classification CA.

2. Interpretation of findings

Both these findings illustrate Kropp et al's (1966) formulation that as the taxonomic level of questions increased so pupils' average performances would decrease.

The lack of significant decrease in quintiles 3, 4 and 5 may just be due to the mathematical influence of the approach of zero as the averages of the questions classified as K show a linear decline whereas those of CA and AS/I decrease on a flattening curve (see table 5.19). In H25 the

slightly higher average for AS classified questions compared with those classified as CA may only be a reflection of the conclusion noted already that the AS classification is dominated by scores from higher ability students while the CA classification contains scores mainly from students of low ability. These factors will affect the result and so Kropp's formulation for teacher classified questions which is confirmed here cannot be rejected in its entirety when pupils classify the questions. Again further study will have to seek to confirm or refute the finding.

Examination of the Methodological Procedures Used

Firstly it must be realised that this study possesses the usual weaknesses that a ny ex-post facto research possesses. Thus it can be regarded as being of an exploratory nature only, and the conclusions must be limited to those of showing relationships. There can be no definite conclusion as to what is cause and what effect, although an inference is drawn that because pupils first wrote the papers and only afterwards answered the questionnaire, having their scripts in front of them while doing so, their classification of the questions would be affected by their ability to answer the question rather than the other way round.

Secondly the study took a cross-section of a number of schools and combined all their data. This resulted in there being two basic styles of resultant classification ie definite and split. In discussing subsequent data it would have been better to have considered only those questions where the resultant classifications were definite.

This would, however, have resulted in discarding too much data. Thus questions classified as split assessments had to be used, although their very indefiniteness of necessity affected the character of the data.

In order to obviate this in future studies one of two procedures may be adopted:

- a. Three or four papers could be set, each answered by eight or more schools, which would mean that instead of having 50 questions to choose from there would be 200 questions, and this should result in sufficient data being left when questions classified as split resultants are discarded.
- b. A better procedure would probably be to keep a careful record of which pupils belong to which class group taught by which teacher. The pupils' classifications would then be compared with their own teacher's classification. This would result in a more meaningful comparison between pupil and teacher classifications.

Thirdly, should a cross-section of schools be taken again then it would perhaps be better to consider carrying out the research later in the year when most classes have completed the syllabus.

Chapter 7

Recommendations for Further Research

1. A relationship between the pupils' classifications of mathematics examination questions and their performance scores has been established. It is important to ascertain which is the independent and which the dependent variable or whether these two variables interact with each other or are acted upon by another factor in order to produce the relationship.
2. As indicated in the previous section pupils' classifications should be compared with their own teacher's classification in order to obtain more definitive conclusions as far as teacher and pupil classification agreement is concerned. If agreement is high then the probability is high that properties applicable to teacher classifications would also be applicable to pupil classifications.
3. It could be of importance to consider whether there is a relationship between the type of error that a pupil makes in a question and his classification of that question. For example, would a conceptual error occur more often in questions classified as knowledge or in higher order questions? Following on this, it could then be of use to ascertain whether the difficulty of a question induces a candidate to make a particular kind of error.
4. A final suggestion for further research is to ascertain whether the ability of a pupil to assess

- a question according to a taxonomic scale will help him in the answering of that question or not. Will the candidate, for example, be prepared to think to greater depth if he can assess what type of question is being asked? Also if he is aware that in a particular examination, questions of a certain taxonomic order predominate, will that help him in his preparation for that examination? The answers to these questions will have an effect on the teaching of Mathematics.

All research into examining must have as its final aim the improvement of the teaching process, otherwise it is an end in itself. The present writer trusts that the establishment of the relationship between a pupil's ability in Mathematics as a whole, as well as his ability to answer the question asked, and his classification of a question taxonomically, will affect the way in which a teacher questions in the classroom because he is more aware of the pupil's perception of what he is being asked.

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APPENDIX A1 : PAPER SET FOR CANDIDATES

Standard 10

Time : 3 hours

MATHEMATICS HIGHER GRADE (FIRST PAPER)

MAY/JUNE 1982

Above each answer write the number of the question.
All necessary work must be shown in its proper place
with the answer.

You will be instructed as to which Question 9 you are
to answer.

QUESTION 1

- a) If $f = \{(1;2);(-3;1);(2;2);(-3;3)\}$:
- state whether f represents a function or not, and state the type of correspondence involved;
 - find a if $f(a) = 2$;
 - find k if $f(-3) = k$. (6)
- b) Without using tables establish which is the greater:
 $\sqrt{3}$ or $\sqrt[3]{5}$? (3)
- c) Draw a sketch graph of the function $y = ax^2 + bx + c$ if $a < 0$, $c > 0$ and the sum of the roots of $ax^2 + bx + c = 0$ is positive. (5)
- d) If $a \frac{m}{n}$ is real what values may a not have if m is odd and n is even? (2)
- e) Sketch the graph of $y = a^x$ if $0 < a < 1$. Indicate any intersection with the axis. (4)
/20/

QUESTION 2

- a) Consider the equations $y = \sqrt{25 - x^2}$ and $y = -x - 1$
- Draw sketch-graphs of these equations on the same system of axes indicating where the graphs intersect the axes. (5)
 - Calculate the co-ordinates of the point(s) where the two graphs intersect. (7)
 - Show algebraically that the point (3;4) lies on the graph of $y = \sqrt{25 - x^2}$ (1)
 - By direct use of symmetry find two other

points on the graph of $y = \sqrt{25 - x^2}$ which are symmetrical to the point (3;4), stating which line of symmetry is being used in each case.

v) Show by shading : $\{(x;y) : -x - 1 \leq y \leq \sqrt{25 - x^2}, x > 0\}$ (4)
/21/

b) Consider $h = \{(x;y) : y = (x - 2)^2\}$

i) Draw a sketch graph of h showing the axis of symmetry and the co-ordinates of the X and Y intercepts. (3)

ii) Now find the defining equation of h^{-1} expressing y in terms of x . (2)

iii) On the same system of axes as in (i) draw a sketch graph of h^{-1} , and of the graph defined by $y = x$ showing the intersection with the axes. (5)

iv) Give the domain of h^{-1} . (1)

v) On the same system of axes draw a sketch graph of $\{(x;y) : xy = 4\}$ (2)

vi) From the graph determine the values of x for which $\frac{4}{x} (x - 2)^2 \leq 0$ (2)
/15/
/36/

QUESTION 3

a) Consider $x^2 + 2mx + m = 0$

i) Solve the equation for x by completing the square. (5)

ii) Hence find for which values of m the roots will be equal. (3)
(8)

b) Solve for x : $\frac{6}{x - 3} \leq x + 2$ (7)

c) Determine without the use of tables the minimum value of y if:

$$(3^y + 2 - 18)(x^2 - 9) \leq 0 \text{ and } -3 < x < 3$$

(NB. The answer is not a rational number) (7)
/22/

QUESTION 4

a) Assuming that the solution to the equation $ax^2 + bx + c = 0$ is given by

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, prove that the sum of the roots of the equation is given by $-\frac{b}{a}$. (You may use another method to prove this result if you so wish). Give the conditions that will make the sum of the roots positive. (5)

b) Prove that the roots of $x^2 - 2x + 3 = 4kx - 6k^2$ are imaginary (non-real) for all real values of k except for one value of k . Give this value of k and state the nature of the roots if k has this value. (10)

c) i) Find the maximum value of the sum of the roots of $\frac{1}{a}x^2 + (a+1)x - b = 0$ (5)

ii) What effect does the result in (i) have on the axis of symmetry of the graph of

$$y = \frac{1}{a}x^2 + (a+1)x - b? \quad (3)$$

(8)
/23/

QUESTION 5

a) Prove that $a^m \cdot a^n = a^{m+n}$, if $a \in \mathbb{R}$, and $m, n \in \mathbb{N}$. (3)

b) Simplify : $\left(27^{\frac{2}{3}} + 9^{\frac{3}{2}}\right) \times (81)^{-\frac{3}{4}}$ (5)

c) Solve for x without using tables in:

i) $9^{2x+1} \times \frac{1}{27^x} = \frac{1}{3}$ (3)

ii) $4^x + 1 + 2^{2x+1} + \left(\frac{1}{4}\right)^{-x} = \frac{7}{16}$ (5)

iii) $\sqrt{x} \cdot 3^{-x} = \frac{1}{3}$ (7)

(15)
/23/

QUESTION 6

a) Calculate the value of $2^{\log_{\frac{1}{2}} 3}$ without the use of tables. (3)

b) Solve for x using tables:

$$\log_{10} x + \log_{10} (x+2) = 0,903 \quad (7)$$

c) i) Prove that $\log_a p = \frac{1}{\log_p a}$ (2)

ii) Hence, or otherwise, solve for x :

$$\log_3 x + 2 \log_x 3 = 3 \text{ without tables.} \quad (7)$$

(9)
/19/

QUESTION 7

- a) The sum of a Geometric Series is given by

$$S_n = \frac{a(1 - r^n)}{1 - r}$$
 where a is the first term, r is the common ratio and n is the number of terms:
- i) What value may r not have in this formula? (1)
 - ii) How would the sum to n terms of a Geometric Series be found if r had this value? $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
- b) Calculate $\sum_{n=1}^{40} (2n - 7)$ (5)
- c) If $4 + 2 + 1 + \dots + 8\left(\frac{1}{2}\right)^n = 7\frac{63}{64}$:
- i) find n (6)
 - ii) determine the limit to which the sum of the series will tend as n tends to infinity. $\begin{pmatrix} 2 \\ 8 \end{pmatrix}$
- d) Calculate the tenth term of the series for which the sum to n terms is $2n^2 - 3n$ (8)
 /24/

QUESTION 8

The three ordered pairs $(-2;-7)$; $(0;1)$ and $(1;2)$ can ALL be elements of ONLY ONE of the following relations:

- $\{(x;y) : y = p^x, p > 0\}$
- $\{(x;y) : y = \log_a x\}$
- $\{(x;y) : y = -\sqrt{r^2 - x^2}\}$
- $\{(x;y) : y = \frac{k}{x}\}$
- $\{(x;y) : y = ax^2 + bx + c\}$
- $\{(x;y) : y = mx + c\}$

(a , b , c , m , p and r are real constants having their usual meanings).

- i) State the 5 relations which CANNOT have all three ordered pairs as elements, giving an algebraic reason for EACH answer. (11)
- ii) Determine the defining equation of the relation which is satisfied by ALL THREE ordered pairs. $\begin{pmatrix} 8 \\ 19 \end{pmatrix}$

QUESTION 9EITHER

- a) Find an equation in the form $ax^2 + bx + c = 0$ if the roots are $x = -p \pm \sqrt{p^2 - q^2}$ (4)
- b) Consider the function defined by $y = a(x + m)^2 + n$. Without the use of formulae prove that:
- i) $x = -m$ is the axis of symmetry of the graph that can be drawn; (7)
 - ii) n is the maximum value if $a < 0$. (3)
- (10)

ORQUESTION 9

Consider $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$

- i) Prove that it is true if $n = 3$. (3)
 - ii) Assuming that it is true for $n = p$, prove that it is true for $n = p + 1$. (6)
 - iii) What does the result in (ii) mean for the sum to 96 terms and the sum to 97 terms in the series given? (2)
 - iv) For what must it now be proved to make it true for all $n \in \mathbb{N}$? (1)
 - v) Calculate $\sum_{n=1}^{400} \frac{1}{(2p-1)(2p+1)}$ (2)
- /14/

TOTAL : 200

APPENDIX A2 : VRAESTEL VIR KANDIDATE

Standaard 10

Tyd : 3 uur

WISKUNDE HOËR GRAAD (EERSTE VRAESTEL)

Mei/Junie 1982

Skryf bo elke antwoord die nommer van die vraag.

Al die nodige werk moet op dié regte plek met u antwoord aangedui word.

U onderwyser sal vir u sê watter Vraag 9 u moet antwoord.

VRAAG 1

- a) As $f = \{(1;2);(-3;1);(2;2);(-3;3)\}$
- dui aan of f 'n funksie is of nie en meld die betrokke tipe afbeelding:
 - bepaal a as $f(a) = 2$;
 - bepaal k as $f(-3) = k$. (6)
- b) Sonder die gebruik van tafels, bepaal watter van die volgende die grootste is:
- $$\sqrt{3} \qquad \sqrt[3]{5} \qquad (3)$$
- c) Trek 'n sketsgrafiek van die funksie gedefinieer deur $y = ax^2 + bx + c$ as $a < 0$, $c > 0$ en die som van die wortels van $ax^2 + bx + c = 0$ 'n positiewe waarde het. (5)
- d) As $a^{\frac{1}{n}}$ reëel is, watter waardes mag a nie hê nie as m onewe en n ewe is. (2)
- e) Skets die grafiek van $y = a^x$ as $0 < a < 1$ is. Dui enige afsnitte met die asse aan. (4)
(20)

VRAAG 2

- a) Beskou die vergelykings $y = \sqrt{25 - x^2}$ en $y = -x - 1$.
- Trek sketsgrafieke van hierdie vergelykings op dieselfde assstelsel en dui aan waar die grafiek die asse sny. (5)
 - Bereken die koördinate van die twee grafieke se kruispunt(e). (7)
 - Bewys algebraïes dat die punt $(3;4)$ op die grafiek van $y = \sqrt{25 - x^2}$ lê. (1)
 - Bepaal twee ander punte op die grafiek $y = \sqrt{25 - x^2}$ deur direkte gebruik van simmetrie.

Noem ook die simmetrie - as wat in elke geval gebruik word.

v) Wys deur arsering : $\{(x;y) : -x - 1 \leq y \leq \sqrt{25 - x^2}, x > 0\}$ (4)
/21/

b) Beskou $h = \{(x;y) : y = (x - 2)^2\}$

i) Trek 'n sketsgrafiek van h en dui die simmetrie-as en die koördinate van die X- en Y- afsnitte aan. (3)

ii) Bepaal die definiërende vergelyking van h^{-1} , met y uitgedruk in x . (2)

iii) Op dieselfde assestelsel as in (i) trek 'n sketsgrafiek van h^{-1} en van die grafiek gedefinieer deur $y = x$ en dui die afsnitte met die asse aan. (5)

iv) Gee die gebied (definisieversameling) van h^{-1} (1)

v) Op dieselfde assestelsel trek 'n sketsgrafiek van $\{(x;y) : xy = 4\}$ (2)

vi) Lei van die grafiek af die waardes van x waarvoor $\frac{4}{x} (x - 2)^2 \leq 0$ (2)
/15/
/36/

VRAAG 3

a) Beskou $x^2 + 2mx + m = 0$

i) Los vir x op deur kwadraatsvoltooiing. (5)

ii) Lei hiervan af vir watter waardes van m die wortels gelyk sal wees. (3)
(8)

b) Los op vir $x : \frac{6}{x-3} \leq x + 2$ (7)

c) Bepaal die minimumwaarde van y sonder die gebruik van tafels as:

$(3^y + 2 - 18)(x^2 - 9) \leq 0$ en $-3 < x < 3$
(L.W. Die antwoord is NIE 'n rasionale getal nie) (7)
/22/

VRAAG 4

a) Aanvaar dat die oplossing van die vergelyking $ax^2 + bx + c = 0$ deur $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gegee

word en bewys dat die som van die wortels van die vergelyking $-\frac{b}{a}$ is. (Enige ander metode mag ook vir hierdie bewys gebruik word). Watter waardes van a en b sal bepaal dat die waarde van die som van die wortels positief sal wees? (5)

- b) Bewys dat die wortels van $x^2 - 2x + 3 = 4kx - 6k^2$ denkbeeldig (nie-reëel) is vir alle reële waardes van k met die uitsondering van een waarde van k . Noem die aard van die wortels as k hierdie waarde het en gee hierdie waarde van k . (10)
- c) i) Bepaal die maksimumwaarde van die som van die wortels van $\frac{1}{a}x^2 + (a+1)x - b = 0$ (5)
- ii) Watter gevolg het die uitslag in (i) op die simmetrie-as van die grafiek gedefinieer deur:
 $y = \frac{1}{a}x^2 + (a+1)x - b$? (3)
 (8)
 /23/

VRAAG 5

- a) Bewys dat $a^m \cdot a^n = a^{m+n}$, as $a \in \mathbb{R}$ en $m, n \in \mathbb{N}$ (3)
- b) Vereenvoudig : $(27^{\frac{2}{3}} + 9^{\frac{3}{2}}) \times (81)^{-\frac{3}{4}}$ (5)
- c) Los op vir x sonder die gebruik van tafels in:
- i) $9^{2x+1} \times \frac{1}{27^x} = \frac{1}{3}$ (3)
- ii) $4^x + 1 + 2^{2x+1} + \left(\frac{1}{4}\right)^{-x} = \frac{7}{16}$ (5)
- iii) $\sqrt[3]{9} \cdot 3^{-x} = \frac{1}{3}$ (7)
 /15/
 /23/

VRAAG 6

- a) Bereken die waarde van $2^{\log_{\frac{1}{2}} 3}$ sonder die gebruik van tafels. (3)
- b) Met die gebruik van tafels los op vir x :
 $\log_{10} x + \log_{10} (x+2) = 0,903$ (7)
- c) i) Bewys dat $\log_a p = \frac{1}{\log_p a}$ (2)
- ii) Deur gebruik te maak van (i), of andersins, los op vir x sonder die gebruik van tafels:
 $\log_3 x + 2 \log_x 3 = 3$ (7)
 (9)
 /19/

VRAAG 7

- a) Die som van n meetkundige reeks word deur
 $s_n = \frac{a(1-r^n)}{1-r}$ gegee, waar a die eerste term, r die gemene verhouding en n die aantal terme is.

- i) Watter waarde kan r in hierdie formule nie aanneem nie? (1)
- ii) Hoe kan die som tot n terme van 'n meetkundige reeks gevind word as r hierdie waarde het? (2)
(3)
- b) Bereken $\sum_{n=1}^{40} (2n - 7)$ (5)
- c) As $4 + 2 + 1 + \dots + 8\left(\frac{1}{2}\right)^n = 7\frac{63}{64}$:
- i) bepaal n ; (6)
- ii) bepaal die limiet waarna die som van die reeks streef as n na oneindig strewe. (2)
(8)
- d) Bereken die tiende term van die reeks waarvan die som tot n terme $2n^2 - 3n$ is. (8)
/24/

VRAAG 8

Die geordende getalleepare $(-2;-7); (0;1); (1;2)$ kan AL DRIE elemente wees van slegs EEN van die volgende ses relasies:

$$\begin{aligned} &\{(x;y) : y = p^x, p > 0\} \\ &\{(x;y) : y = \log_a x\} \\ &\{(x;y) : y = \sqrt{r^2 - x^2}\} \\ &\{(x;y) : y = \frac{k}{x}\} \\ &\{(x;y) : y = ax^2 + bx + c\} \\ &\{(x;y) : y = mx + c\} \end{aligned}$$

(a, b, c, m, p en r is reële konstantes wat al hulle gewone betekenisse het)

- i) Noem die VYF relasies wat NIE al drie die geordende getalleepare as elemente kan hê nie. Gee 'n algebraïese rede vir ELKE antwoord. (11)
- ii) Bepaal die definiërende vergelyking van die relasie wat deur AL DRIE geordende getalleepare bevredig word. (8)
/19/

OF

VRAAG 9

- a) Bepaal 'n vergelyking in die vorm $ax^2 + bx + c = 0$ as die wortels $x = -p \pm \sqrt{p^2 - q^2}$ is. (4)
- b) Beskou die funksie gedefinieer deur $y = a(x + m)^2 + n$. Sonder die gebruik van formules bewys dat:

i) $x = -m$ die simmetrie-as van die grafiek wat getrek kan word is; en

ii) n die maksimumwaarde is as $a < 0$. (10)
/14/

OF

VRAAG 9

Beskou $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$

i) Bewys dat dit waar is vir $n = 3$ (3)

ii) Gestel dat dit waar is vir $n = p$, bewys dat dit ook waar is vir $n = p + 1$ (6)

iii) Wat is die betekenis van die resultaat in (ii) in verband met die som tot 96 terme en die som tot 97 terme in die reeks wat gegee word? (2)

iv) Waarvoor moet dit nou waar bewys word om dit waar vir alle $n \in \mathbb{N}$ te bewys? (1)

v) Bereken $\sum_{p=1}^{400} \frac{1}{(2p-1)(2p+1)}$ (2)
/14/

TOTAAL : 200

APPENDIX A3MEMORANDUM OF MARKING

STD 10 EXAMINATION PAPER / ST 10 EKSAMENVRAESTEL
 MATHEMATICS HIGHER GRADE / WISKUNDE HOËR GRAAD
 FIRST PAPER / EERSTE VRAESTEL
 MAY - JUNE 1982 / MEI - JUNIE 1982

Question 1 / Vraag 1

a (i) Not a function / geen funksie
 Many-to-many / Meer-meerduidelig

(ii) $a = 1; 2$

(iii) $k = 1; 3$

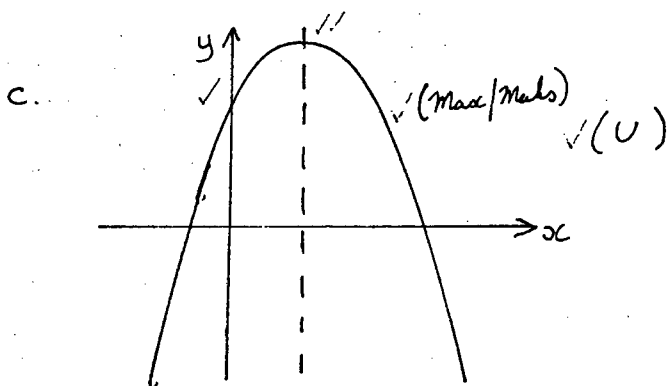
(6)

b. $\sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}$

$\sqrt[3]{5} = \sqrt[6]{5^2} = \sqrt[6]{25}$

$\therefore \sqrt{3} > \sqrt[3]{5}$

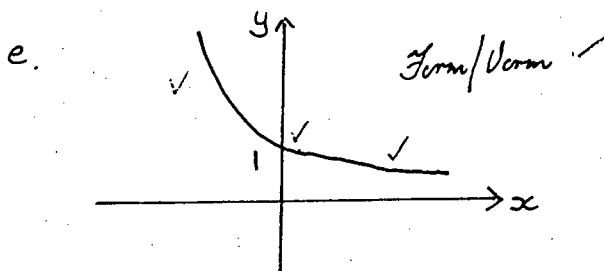
(3)



(5)

d. a must not be < 0

(2)



(4)[20]

Question 2/Vraag 2

a i) semi - 0 ✓

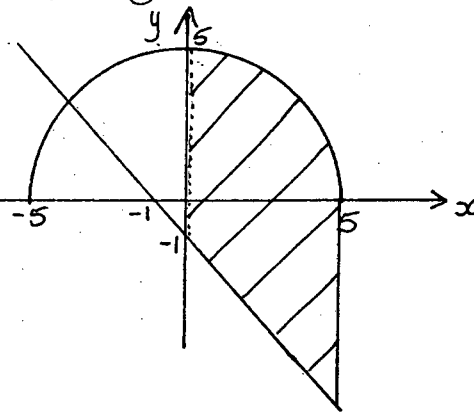
✓

✓

str line / regst lyn

x-int / afsnit ✓

y-int / afsnit ✓



5

(ii) $y = -x - 1 \dots \textcircled{1}$ $y = \sqrt{25 - x^2} \dots \textcircled{2}$

$$\therefore -x - 1 = \sqrt{25 - x^2} \quad \checkmark$$

$$\therefore x^2 + 2x + 1 = 25 - x^2 \quad \checkmark$$

$$\therefore 2x^2 + 2x - 24 = 0 \quad \checkmark$$

$$\therefore x^2 + x - 12 = 0$$

$$\therefore (x + 4)(x - 3) = 0 \quad \checkmark$$

$$\therefore x = -4 \text{ or / of } x = 3 \quad \checkmark$$

Subst / ystel in ① $y = 3$ or / of $y = -4 \quad \checkmark$

Co-ords / Ko-ords = $(-4; 3) \quad \checkmark$

7

(iii) If / As $x = 3$ $\sqrt{25 - x^2} = \sqrt{25 - 9} = 4 \quad \checkmark$

$\therefore (3; 4)$ is on / lê op f

(iv) $(-3; 4) \quad \checkmark$ Y-axis / Y-as ✓

$(4; 3) \quad \checkmark$ $y = x \quad \checkmark$

4

(v) On graph / Op grafiek

/// ✓

Under / Onder $\cap \quad \checkmark$

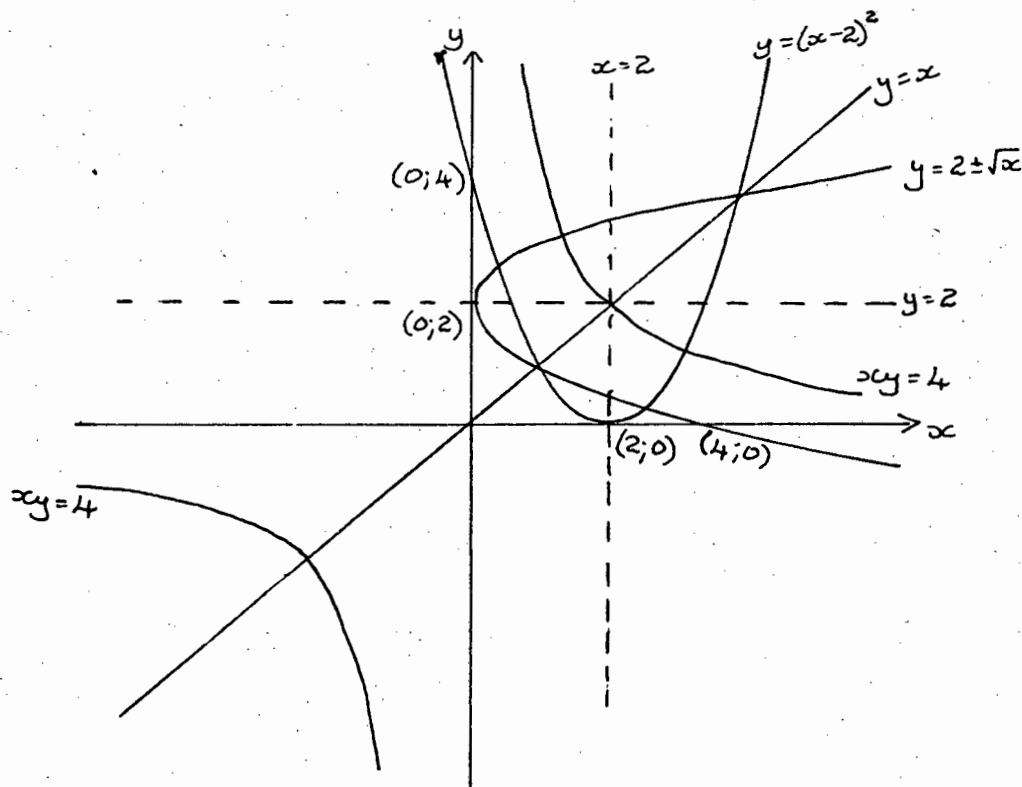
Above / Bo $\backslash \quad \checkmark$

Left of / Links van $x = 5 \quad \checkmark$

4

(21)

b



(i) On graph / \cup ✓
 On grafiek x & y int / afsnitte ✓ (Notation / Notasie -1)
 Axis of sym / sim-as ✓ 3

(ii) $h^{-1} : x = (y-2)^2$ ✓
 $\therefore \pm\sqrt{x} = y-2$
 $\therefore 2 \pm \sqrt{x} = y$ ✓ 2

(iii) On graph / On grafiek : \subset ✓
 Axis of sym / sim-as > 0 ✓
 x and y int / afsnitte ✓
 $y = x$ ✓
 int with $y = x$ / afsnitte met $y = x$ ✓ 5

(iv) $X_{h^{-1}} = \{x : x \geq 0\}$ ✓ 1

(v) On graph / On grafiek ✓
 ✓ 2

(vi) $x < 0$; $x = 2$ ✓ 2

(15)

[36]

Question 3/Vraag 3

a (i)

$$x^2 + 2mx + m = 0$$

$$\therefore x^2 + 2mx = -m$$

$$\therefore x^2 + 2mx + m^2 = m^2 - m$$

$$\therefore (x+m)^2 = m^2 - m$$

$$\therefore x+m = \pm \sqrt{m^2 - m}$$

$$\therefore x = -m \pm \sqrt{m^2 - m} \quad \rightarrow$$

(ii)

$$m^2 - m = 0 \quad \checkmark \text{ for equal roots / vir gelyke wortels}$$

$$\therefore m(m-1) = 0 \quad \checkmark$$

$$\therefore m = 0 \text{ or / of } m = 1 \quad \checkmark$$

If use Δ / As Δ gebruikte Max/Min's \checkmark

(8)

b

$$\frac{6}{x-3} \leq x+2$$

$$\therefore \frac{6}{x-3} - x - 2 \leq 0 \quad \checkmark$$

$$\therefore \frac{6 - x^2 + 3x - 2x + 6}{x-3} \leq 0 \quad \checkmark$$

$$\therefore \frac{-x^2 + x + 12}{x-3} \leq 0$$

$$\therefore \frac{(-x+4)(x+3)}{x-3} \leq 0 \quad \checkmark$$



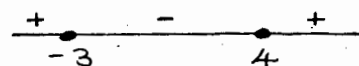
$$\therefore -3 \leq x < 3; x \geq 4 \quad \rightarrow$$

$$\text{If/as } 6 \leq (x+2)(x-3)$$

$$\therefore 6 \leq x^2 - x - 6$$

$$\therefore 0 \leq x^2 - x - 12 \quad \checkmark$$

$$\therefore 0 \leq (x-4)(x+3) \quad \checkmark$$



$$\therefore x \leq -3 \text{ or } x \geq 4 \quad \checkmark$$

(7)

$$c. (3^{y+2} - 18)(x^2 - 9) \leq 0 \quad -3 < x < 3$$

$$\therefore 3^{y+2} - 18 \geq 0 \quad \checkmark \checkmark$$

$$\therefore 3^y \cdot 9 \geq 18$$

$$\therefore 3^y \geq 2 \quad \checkmark$$

$$\therefore y \geq \log_3 2 \quad \checkmark \checkmark$$

$$\therefore \text{Min value / waarde} = \log_3 2 \quad \checkmark$$

(7)

[22]

Question 4 / Vraag 4

a. Sum of roots / Som van wortels $= \frac{-b+\sqrt{\Delta}}{2a} + \frac{-b-\sqrt{\Delta}}{2a} \checkmark$
 $= -\frac{2b}{2a} \checkmark$
 $= -\frac{b}{a} \checkmark$

For sum / Vir som > 0 $a > 0; b < 0$ or / of $a < 0; b > 0$
 or / a and b opp in sign /
 $a = b$ verskillende tekens (5)

b. $x^2 - 2x + 3 = 4kx - 6k^2$
 $\therefore x^2 - 2x - 4kx + 3 + 6k^2 = 0$
 $\therefore x^2 - (2+4k)x + 3+6k^2 = 0 \checkmark$
 $\Delta = (2+4k)^2 - 4(3+6k^2) \checkmark$
 $= 4+16k+16k^2 - 12-24k^2 \checkmark$
 $= -8k^2 + 16k - 8 \checkmark$
 $= -8(k^2 - 2k + 1)$
 $= -8(k-1)^2 \checkmark$

$\Delta < 0$ unless / tensy $k=1 \checkmark$
 \therefore Roots imaginary unless $k=1$ / Wortels denkbeeldig tensy $k=1 \checkmark$
 If $k=1$ roots are real, rational and equal /
 As $k=1$ wortels is reëel, rasionaal en gelyk. (10)

c.(i) $\frac{1}{a}x^2 + (a+1)x - b = 0$
 Sum of roots / Som v.d. wortels $= -\frac{a+1}{\frac{1}{a}} \checkmark$
 $= -a(a+1)$
 $= -a^2 - a \checkmark$

Max value / Maks waarde $= -\frac{\Delta}{4a} = -\frac{1}{-4} = \frac{1}{4} \checkmark$

(ii) Axis of symmetry not to right of $x = \frac{1}{8} \checkmark$
 Simmetrie-as nie regs van $x = \frac{1}{8}$

(8)

[23]

Question 5/Varag 5

$$\begin{aligned}
 a. \quad a^m \cdot a^n &= (a \cdot a \cdot a \dots \text{to } m \text{ factors})(a \cdot a \cdot a \dots \text{to } n \text{ factors}) \checkmark \\
 &= a \cdot a \cdot a \dots \text{to } (m+n) \text{ factors} \checkmark \\
 &\xrightarrow{= a^{m+n}} \checkmark
 \end{aligned}
 \tag{3}$$

$$\begin{aligned}
 b. \quad (27^{\frac{2}{3}} + 9^{\frac{3}{2}}) \times 81^{-\frac{3}{4}} &= ((3^3)^{\frac{2}{3}} + (3^2)^{\frac{3}{2}}) \times (3^4)^{-\frac{3}{4}} \\
 &= (9 + 27) \times \frac{1}{27} \checkmark \\
 &= \frac{36}{27} \checkmark \\
 &\xrightarrow{= \frac{4}{3}} \checkmark
 \end{aligned}
 \tag{5}$$

$$\begin{aligned}
 c.(i) \quad 9^{2x+1} \times \frac{1}{27^x} &= \frac{1}{3} \\
 (3^2)^{2x+1} \times \frac{1}{3^{3x}} &= \frac{1}{3} \\
 \therefore 3^{4x+2} \cdot 3^{-3x} &= 3^{-1} \checkmark \\
 \therefore 3^{x+2} &= 3^{-1} \checkmark \\
 \therefore \underline{x = -3} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad 4^{x+1} + 2^{2x+1} + \left(\frac{1}{4}\right)^{-x} &= \frac{7}{16} \\
 \therefore 2^{2x+2} + 2^{2x+1} + 2^{2x} &= \frac{7}{16} \\
 \therefore 2^{2x}(2^2 + 2 + 1) &= \frac{7}{16} \\
 \therefore 2^{2x} \cdot 7 &= \frac{7}{16} \\
 \therefore 2^{2x} &= \frac{1}{16} \checkmark \\
 \therefore 2^{2x} &= 2^{-4} \\
 \therefore \underline{x = -2} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad \sqrt{9} \cdot 3^{-x} &= \frac{1}{3} \\
 \therefore 3^{\frac{2}{x}} \cdot 3^{-x} &= 3^{-1} \checkmark \\
 \therefore \frac{2}{x} - x &= -1 \checkmark \\
 \therefore 2 - x^2 &= -x \\
 \therefore 0 &= x^2 - x - 2 \checkmark \\
 \therefore 0 &= (x-2)(x+1) \checkmark \\
 \therefore x &= 2 \checkmark \text{ or } x = -1 \checkmark \\
 \therefore \underline{x = 2} \checkmark
 \end{aligned}$$

(15)

[23]

Question 6/Vraag 6

a. Laat/Let $\log_{\frac{1}{2}} 3 = x$
 $\therefore 3 = \left(\frac{1}{2}\right)^x$ ✓
 $\therefore \frac{1}{3} = 2^x$ ✓
 $\therefore \frac{1}{3} = 2^{\log_{\frac{1}{2}} 3}$ ✓

OR/OF Let/Laat $2^{\log_{\frac{1}{2}} 3} = x$
 $\therefore \log_{\frac{1}{2}} 3 = \log_2 x$ ✓
 $\therefore \log_2 \frac{1}{3} = \log_2 x$ ✓
 $\therefore \frac{1}{3} = x$ ✓ (3)

b. $\log_{10} x + \log_{10} (x+2) = 0,903$

$\therefore \log x(x+2) = \log 8$ ✓

$\therefore x(x+2) = 8$ ✓

$\therefore x^2 + 2x - 8 = 0$ ✓

$\therefore (x+4)(x-2) = 0$ ✓

$\therefore x = -4$ or/of $x = 2$ ✓

$\therefore \underline{x = 2}$ ✓ (7)

c (i) $\log_a p = \frac{\log p}{\log a} = 1 \div \frac{\log a}{\log p} = \log_p a$ (2)

(ii) $\log_3 x + \frac{2}{\log_3 x} = 3$ OR/OF $\frac{\log x}{\log 3} + \frac{2 \log 3}{\log x} = 3$ ✓

$\therefore (\log_3 x)^2 + 2 = 3 \log_3 x$

$\therefore (\log x)^2 + 2(\log 3)^2 = 3 \log 3 \log x$

$\therefore (\log_3 x)^2 - 3 \log_3 x + 2 = 0$ ✓

$\therefore (\log x)^2 - 3 \log x \log 3 + 2(\log 3)^2 = 0$ ✓

$\therefore (\log_3 x - 1)(\log_3 x - 2) = 0$

$\therefore (\log x - 2 \log 3)(\log x - \log 3) = 0$ ✓

$\therefore \log_3 x = 1$ or/of $\log_3 x = 2$ ✓

$\therefore \log x = 2 \log 3$ or/of $\log x = \log 3$ ✓

$\therefore \underline{x = 3}$ or/of $x = 3^2$ ✓

$\therefore x = 3^2$ or/of $\underline{x = 3}$ ✓

$\underline{x = 9}$ ✓

$\underline{x = 9}$ ✓

(7)

[19]

Question 7 / Vraag 7

a. (i) $r = 1$ ✓

(ii) If/as $r = 1$ $S_n = an$ ✓ (3)

b. $\sum_{n=1}^{40} (2n-7) = -5-3-1+\dots$ OR/OF $\sum_{n=1}^{40} (2n-7) = -5-3-\dots+73$ ✓

$S_n = \frac{n}{2} [2a + (n-1)d]$ ✓

$S_n = \frac{n}{2} (a+l)$ ✓

$S_{40} = \frac{40}{2} [-10 + 39(2)]$ ✓

$= \frac{40}{2} (-5+73)$

$= 20(68)$

$= 20(68)$

$= 1360$ ✓

$= 1360$ ✓

(5)

c (i) $S_n = \frac{a(1-r^n)}{1-r}$

$\therefore 7 \frac{63}{64} = \frac{4(1-(\frac{1}{2})^n)}{1-\frac{1}{2}}$ ✓

$\therefore \frac{511}{64} = 8(1-(\frac{1}{2})^n)$

$\therefore \frac{511}{512} = 1-(\frac{1}{2})^n$

$\therefore (\frac{1}{2})^n = \frac{1}{512}$ ✓

$\therefore (\frac{1}{2})^n = (\frac{1}{2})^9$ ✓

$\therefore n = 9$ ✓

(ii) Limit / Limiet = 8 ✓

OR/OF $S_\infty = \frac{a}{1-r} = \frac{4}{1-\frac{1}{2}} = 8$ ✓

(8)

d. $S_{10} = 2(100) - 3(10) = 170$ ✓

$S_9 = 2(81) - 3(9) = 162 - 27 = 135$ ✓

$T_{10} = S_{10} - S_9 = 35$ ✓

OR/OF $S_n = 2n^2 - 3n$

$S_1 = 2 - 3 = -1 \therefore a = -1$ ✓

$S_2 = 8 - 6 = 2 \therefore T_2 = 3$ ✓

$S_3 = 18 - 9 = 9 \therefore T_3 = 7$ ✓

$T_n = a + (n-1)d$

$T_{10} = -1 + 9(4)$ ✓

$= 35$ ✓

(8) [24]

Question 8/Vraag 8

(i) $\left\{ \begin{array}{ll} y = p^x & y > 0 \\ y = \log_a x & x > 0 \\ y = -\sqrt{r^2 - x^2} & y > 0 \text{ or/ of } r \text{ constant/konstant} \\ y = \frac{k}{x} & x \neq 0 \text{ or/ of } k \text{ constant/konstant} \\ y = mx + c & \text{gradient constant/konstant} \end{array} \right. \quad (11)$

(ii) $y = ax^2 + bx + c$

Using/gebruik $(0; 1) \quad 1 = c \quad \checkmark$

Using/gebruik $(1; 2) \quad 2 = a + b + 1 \quad \checkmark \dots \textcircled{1}$

Using/gebruik $(-2; -7) \quad -7 = 4a - 2b + 1 \quad \checkmark \dots \textcircled{2}$

$\textcircled{1} \times 2 \quad 4 = 2a + 2b + 2 \quad \checkmark \dots \textcircled{3}$

$\textcircled{2} + \textcircled{3} \quad -3 = 6a + 3$

$-6 = 6a \quad \checkmark$

$-1 = a \quad \checkmark$

Stel/subst in $\textcircled{1}$

$2 = -1 + b + 1$

$\therefore 2 = b \quad \checkmark$

Defining equation/Definiërende verg $y = -x^2 + 2x + 1 \quad \checkmark$ (8)
[19]

Question 9/Vraag 9

a. $x = -p \pm \sqrt{p^2 - q^2}$
 $\therefore x + p = \pm \sqrt{p^2 - q^2} \quad \checkmark$
 $\therefore x^2 + 2px + p^2 = p^2 - q^2 \quad \checkmark$
 $\therefore x^2 + 2px + q^2 = 0 \quad \checkmark$
 OR/OR $x^2 - (-p + \sqrt{p^2 - q^2} - p - \sqrt{p^2 - q^2})x + (-p + \sqrt{p^2 - q^2})(-p - \sqrt{p^2 - q^2}) = 0$
 $\therefore x^2 - (-2px) + (p^2 - (p^2 - q^2)) = 0$
 $\therefore x^2 + 2px + q^2 = 0 \quad \checkmark \quad (4)$

(b) $y = a(x+m)^2 + n$

(i) Subst/Stel $-m+h$ and/en $-m-h$ for/ver x

If/as $x = -m+h$

then/dan $y = a(-m+h+m)^2 + n = ah^2 + n$

If/as $x = -m-h$

then/dan $y = a(-m-h+m)^2 + n = ah^2 + n$

y-values same / y-waardes gelyk } ✓

∴ Axis of sym / Sim-as is $x = -m$ } ✓ (7)

(ii) $a(x+m)^2 \leq 0$ for all / vir alle $x \in \mathbb{R}$

∴ Max value / Maks waarde van $a(x+m)^2$ is 0 if/as $x = -m$

..... $a(x+m)^2$ is 0 if/as $x = -m$ (3) (10) [14]

OR/OF

Question 9 / Vraag 9

(i) If/as $n=3$ LHS/LK = $\frac{1}{3} + \frac{1}{15} + \frac{1}{35} = \frac{35+7+3}{105} = \frac{45}{105} = \frac{3}{7}$ ✓
RHS/RK = $\frac{3}{6+1} = \frac{3}{7}$ ✓ 3

∴ True for $n=3$ / Waar vir $n=3$

(ii) RTP / Te bewys $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2p+1)(2p+3)} = \frac{p+1}{2p+3}$ ✓

LHS/LK = $\frac{p}{2p+1} + \frac{1}{(2p+1)(2p+3)}$
= $\frac{p(2p+3) + 1}{(2p+1)(2p+3)}$ ✓
= $\frac{2p^2 + 3p + 1}{(2p+1)(2p+3)}$
= $\frac{(2p+1)(p+1)}{(2p+1)(2p+3)}$ ✓
= $\frac{p+1}{2p+3}$ ✓

(iii) If true for $n=96$, true for $n=97$ } ✓
As waar vir $n=96$, waar vir $n=97$ } 2

(iv) For $n=1$ / Vir $n=1$ ✓ 1

(v) $\sum_{p=1}^{400} \frac{1}{(2p-1)(2p+1)} = \frac{p}{2p+1} = \frac{400}{801}$ ✓ 2

[14]
[TOTAL 200]

APPENDIX B1

Letter to Teachers. Instructions to be given by Teachers to Candidates.
Teachers Questionnaire and Instructions for its completion.

Dear Colleagues

I should like to thank you for your help in my research.

It would be appreciated if you would assess each of the questions in the examination paper according to the scale indicated in the questionnaire by circling Yes or No in the appropriate places on the Questionnaire Reply Sheets. Kindly place the word "Teacher" in the space allocated for Student's Number. Please assess all the questions.

Attached are:

1. the questionnaire itself, instructions accompanying it, and the questionnaire reply sheets; and
2. instructions to be given to the pupils for the completion of their questionnaires. A spare copy of the students' questionnaire will also be given to you.

Please keep a list of the student numbers allocated to each pupil.

Please inform the students at the start of the examination paper which Question 9 they are to answer.

Once again my grateful thanks.

D. A. Norton

D. A. NORTON.

INSTRUCTIONS TO BE GIVEN TO THE PUPILS FOR THEIR
COMPLETION OF THE QUESTIONNAIRE

Two or three days before the examination period begins

Please inform the pupils about the following:

"Your Mathematics Higher Grade First Paper Examination has been set by a former Senior Certificate Examiner who will also be marking the scripts. When you have completed writing the paper you will be required to answer a questionnaire which will take half an hour to complete. In order that you may remain anonymous as far as the examiner is concerned you will be issued with examination numbers which you are to place on your paper and the questionnaire answer sheet. These numbers will apply only to Mathematics Higher Grade Paper 1. You will be required to use a pencil when completing the questionnaire, which is concerned with the questions on the examination paper. You will receive your examination scripts after the examination in the normal way."

At the start of the examination

The pupils must write their examination number on their papers, and should staple the pages of their scripts together.

At the end of the examination

1. Tell pupils to put away everything except the question-paper, their answer script and a pencil. No communication may be allowed between the pupils.
2. Hand out the questionnaires and the questionnaire reply sheets informing the pupils that the fact that half have reply sheet 2 on top of reply sheet 1 is deliberate. They are not to separate the sheets and are not to start replying to the questionnaire, before they are told to do so. Ask them to read the first page at least.
3. When replying to the questionnaire they must consult the examination paper, and may look at their answer script if they so wish.
4. Read through the questionnaire and the pupils' instructions answering any queries which pupils may have.
5. Work through Question 1(a)(i) and 5(a) together with the pupils.
6. They may now start. The pupils must answer whichever reply sheet appears on top. They are to reply to that Question 9 which applies to them.

- 7.. The pupils must hand in their examination scripts, examination papers, questionnaires and completed questionnaire reply sheets. The examination question papers will be returned with their scripts.

QUESTIONNAIRE TO BE ANSWERED BY THE TEACHER ON THE
MATHEMATICS HIGHER GRADE EXAMINATION PAPER SET

PART I

- A. Do you think that the pupils have seen this type of question before (ie were the words used to describe what they had to do the same as in classwork questions?) YES/NO
- B. If you answered YES to A would the pupils have found the algorithmic processes:
- i) easy? YES/NO
- ii) of reasonable difficulty or higher? YES/NO
- C. If you answered NO to A i.e. the question was of a type you consider they have not worked at before:
- i) would it have been easy for them to make the connection between their knowledge and the new situation? YES/NO
- ii) would they find that they had to analyse the question carefully and that only after a lot of thought have been able to bring all their knowledge together to answer the question, even though they would recognise certain aspects that were needed in the solution straightaway? YES/NO
- iii) would they find that the question asked something completely different from what they are used to? YES/NO

PART II

A pass in Higher Grade Mathematics is virtually a prerequisite for further study at most universities. Considering this, in what proportion should the marks allocated in an examination paper be divided so as to cover the abilities as given below:

- A. Testing Knowledge
i.e. the candidate should have covered this type of question before.
- A1 Knowledge questions using the simplest algorithmic process/es
- A2 Knowledge questions using more complex algorithmic process/es
- B. Testing abilities which require the application of knowledge to a new situation
i.e. it is likely that the candidates would not have seen aspects of the question before
- B1 Ability to make an easy connection between knowledge candidate has and the new situation

- B2 Ability to analyse a question carefully with much thought prior to using synthesis to present a solution.
- B3 Ability to deal with a question which is completely different to the candidate's experience.

Please enter the percentage of the total marks of a paper that you would allocate to each ability type as listed above.

A1	%	B1	%
A2	%	B2	%
		B3	<u>%</u>
		TOTAL	100%

INSTRUCTIONS FOR COMPLETING TEACHER QUESTIONNAIRE
REPLY SHEET

1. In each line
 - a) If you circle YES to A reply to B
 If you circle NO to A reply to C
- NB Circle B OR C NOT both
- b) In B circle either YES to B(i) and NO to B(ii)
 or NO to B(i) and YES to B(ii)
 In C circle YES to only ONE of C(i), C(ii),
 C(iii)
 circle NO to the other TWO
2. Example of Reply Sheet

Assess- ment Sym- bol Quest- ion	A	B		C		
		(i)	(ii)	(i)	(ii)	(iii)
15(a)	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No
(b)	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No
(c)	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No
(d)	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No
(e)	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No

APPENDIX B2

Brief aan Onderwysers. Instruksies wat aan die
Kandidate deur die Onderwyser gegee moet word.
Onderwyser se Vraelys en Instruksies vir die Voltooi-
ing daarvan.

Geagte Kollegas

Eerstens wil ek u bedank vir al u hulp met my navorsing.

Ek sal dit waardeer as u elk van die vrae in die eksamen vraestel sal beoordeel volgens die skaal wat in die vraelys gegee word. U doen dit deur JA of NEE te omkring in die gepaste plek op die vraelys-antwoordvel. Skryf asseblief die woord "Onderwyser" neer in die plek aangewys vir die leerling se nommer. Beoordeel al die vrae asseblief.

Aangeheg is:

1. die vraelys self, instruksies wat daarmee gepaard gaan en die vraelysantwoordvelle; en
2. instruksies wat aan die leerlinge gegee moet word sodat hulle hul vraelyste kan voltooi. n Ekstra kopie van die studente-vraelys sal aan u gegee word.

U moet asseblief n lys van die studentennommers wat u aan elke leerling gee behou.

U moet die leerlinge by die begin van die eksamen asseblief meedeel watter vraag 9 hulle moet beantwoord.

By voorbaat dankie

Die uwe



D.A. NORTON

INSTRUKSIES WAT AAN LEERLINGE GEGEE MOET WORD VIR DIE VOLTOOIING VAN HUL VRAELYS

Twee of drie dae voor die eksamens begin

Stel asseblief die leerlinge in kennis van die volgende:

"U Wiskunde Hoër Graad Eerste Vraestel eksamen is deur 'n gewese Senior Sertifikaat eksaminator opgestel en hy sal die skrifte ook nasien. Wanneer u die eksamen klaar afgelê het, moet u 'n vraelys voltooi. Die voltooiing daarvan sal sowat 'n halfuur duur. Sodat u anoniem kan bly wat die eksaminator betref, sal eksamennommers, wat u op beide u skrif en die vraelys-antwoordvel moet skrywe, aan u gegee word. Hierdie nommers is beslis net vir die Wiskunde Hoër Graad Vraestel I eksamen bedoel. U moet 'n potlood gebruik met die voltooiing van die vraelys wat oor die vrae in die eksamenvraestel handel. U sal soos gewoonlik u eksamenskrifte terugkry na die eksamen."

By die begin van die Eksamen

Die leerlinge moet hulle eksamennommers op hulle eksamenantwoordskrifte neerskryf, en moet asseblief die velle van die skrifte kram wanneer hulle klaar is.

Aan die Einde van die Eksamen

1. Vertel leerlinge dat hulle alles behalwe die vraestel, hul skrifte en 'n potlood moet verwyder. LW. Daar moet geen kommunikasie tussen leerlinge wees nie.
2. Deel die vraelyste en die vraelysantwoordvelle uit. Terwyl dit gedoen word vertel die leerlinge dat die feit dat die helfte van die leerlinge antwoordvel 2 bo-op antwoordvel 1 het, met opset gedoen is. Hulle moet nie die velle skei nie en moet nie die vraelys begin beantwoord voordat hulle opdrag daarvoor kry nie. Vra hulle om minstens die eerste bladsy te lees.
3. Terwyl hulle die vraelys beantwoord, moet hulle die eksamenvraestel bestudeer, en hulle mag ook hul antwoordskrifte bekyk as hulle wil.
4. Lees die vraelys en die leerlinge se instruksies en beantwoord enige vrae wat die leerlinge mag vra.
5. Werk deur Vraag 1(a)(i) en 5(a) saam met die leerlinge sodat hulle kan sien hoe om die vraelys te voltooi.
6. Hulle mag nou begin. Die leerlinge moet met die antwoordvel wat bo-op is begin en moet die Vraag 9 beantwoord wat op hulle van toepassing is.
7. Die leerlinge moet hul eksamenskrifte, eksamen-

· vraestelle die vraelyste en voltooië vraelys-
antwoordvelle inhandig. Die eksamenvraestelle
sal aan hulle teruggestuur word wanneer hulle
hul skrifte terugkry.

Na die Vraelys voltooi is

Gee al die skrifte en vraelyste ens. aan u senior
wiskunde onderwyser asseblief.

VRAELYS OOR DIE WISKUNDE HOËR GRAAD EKSAMEN VRAESTEL
WAT GESTEL IS WAT DEUR DIE ONDERWYSERS VOLTOOI MOET WORD

DEEL I

- A. Dink u dat die leerlinge hierdie soort vraag tevore gesien het (d.w.s. was die woorde wat gebruik is dieselfde of amper dieselfde as dié wat hy in klaswerkvrae teëgekom het?)
- B. As u JA in A geantwoord het, sou die leerlinge die algoritmiese prosesse:
 - i) Maklik vind?
 - ii) van redelike moeilikheidsgraad of hoër vind?
- C. As u NEE in A geantwoord het, d.w.s. die vraag is van so n tipe dat u meen dat hulle hierdie soort vraag nog nie tevore teëgekom het nie?
 - i) Sou dit vir hulle maklik gewees het om die verbinding tussen hulle kennis en die nuwe situasie in te sien?
 - ii) Sou hulle gevind het dat hulle die vraag sorgvuldig moes ontleed en dat dit net was nadat hulle baie nagedink het, dat hulle al hul kennis sou kon saambring om die vraag te beantwoord, alhoewel hulle sekere aspekte wat in die antwoord gebruik moes word, gou sou herken?
 - iii) Sou hulle vind dat die vraag iets heeltemal anders is as die soort vraag waaraan hulle gewoond is?

DEEL II

Om Hoër Graad Wiskunde te slaag, is feitlik 'n nood-saklike vereiste vir verdere studies in die vak aan universiteite. As dit in ag geneem word, in watter verhouding sou u die punte in 'n eksamenvraestel toeken om die volgende bekwaamhede wat hieronder gegee word te dek?

- A. Die toetsing van kennis d.w.s. die kandidaat moes hierdie soort vraag reeds teëgekom het.
 - A1 Kennisvrae wat die eenvoudigste algoritmiese proses/se vereis.
 - A2 Kennisvrae wat meer ingewikkelde algoritmiese proses/se vereis
- B. Die toetsing van bewaamhede wat die toepassing van kennis in 'n nuwe situasie behels, d.w.s. dit is waarskynlik dat die kandidate sekere aspekte van die vraag nog nie gesien het nie.

- B1 Vermoë om 'n maklike verbinding tussen die kennis wat kandidate het in die nuwe situasie in te sien.
- B2 Vermoë om 'n vraag sorgvuldig te analiseer en nadat die kandidate goed nagedink het, om sintese te gebruik om 'n oplossing te verkry.
- B3 Vermoë om 'n vraag wat verskillend is van aard, van die kandidaat se ondervinding te beantwoord.

Skryf asseblief die persentasie van die totale punte van 'n vraestel wat aan elke bekwaamheid tipe soos hierbo gevind sal toewys, hieronder neer.

A1	%	B1	%
A2	%	B2	%
		B3	%
			<hr/>
			100%

INSTRUKSIES VIR DIE VOLTOOIING VAN DIE ONDERWYSER SE VRAELYSANTWOORDVEL

1. In elke ry:

a) As u JA vir A omkring antwoord B
 As u NEE vir A omkring antwoord C
 LW Omkring in OF B OF C, NIE in beide nie.

b) IN B omkring of JA vir B(i) en NEE vir B(ii)
 of NEE vir B(i) en JA vir B(ii)

In C omkring JA vir net EEN van C(i), C(ii),
 C(iii)
 omkring NEE vir die ander TWEE

2. Voorbeeld van Antwoordvel

Beoorde- ling sim- bool Vraag	A	B		C		
		(i)	(ii)	(i)	(ii)	(iii)
15(a)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(b)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(c)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(d)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(e)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee

APPENDIX B3Letter to Candidates. Student Questionnaire and
Instructions for its Completion

To the Student

In the accompanying questionnaire you are being asked your assessment of the questions you have just answered in the examination paper.

All you have to do is to circle Yes or No in the appropriate place on the Questionnaire Reply Sheets. Use a pencil for the circling.

Please refer to the examination paper as you reply to the questionnaire, and answer as carefully as you can, but do not spend too much time on each response.

Please make sure that you understand the Questionnaire, especially the distinctions between each aspect.

Please follow the instructions of how to fill in the Questionnaire Reply Sheet carefully and also consider the examples given.

Thank you for your help.



D. A. NORTON

QUESTIONNAIRE TO BE COMPLETED BY PUPILS AFTER HAVING
WRITTEN THE EXAMINATION PAPER IN MATHEMATICS HIGHER
GRADE PAPER I

- NB: 1. The word "question" in each case refers to the questions in the examination paper.
2. Circle YES or NO in the appropriate place on Questionnaire Reply Sheet.

Assessment Symbol

- A. Have you seen this type of question before (i.e. were the words used to describe what you had to do and the type of problem the same as questions which you have already done in class?)
- B. If you answered YES to A
- i) Did you learn how to answer this type of question well?
 - ii) Did you find the working-out easy?
- C. If you answered NO to A i.e. the question was of a type you have not worked at before:
- i) Was it easy to make the connection between your knowledge and the new situation?
 - ii) Did you find that you had to analyse the question carefully and that it was only after a lot of thought that you could bring all your knowledge together to answer the question even though you recognised certain aspects that were needed in the solution straightaway?
 - iii) Did you find that the question asked something completely different from what you are used to?

Instructions for Completing Questionnaire Reply Sheet

1. Enter your student's number.
 2. In each row
 - a) If you circle YES to A reply only to B
If you circle NO to A reply only to C
- NB Circle in B or C NOT both
- b) In B circle both B(i) and B(ii)
In C circle YES to only ONE of C(i), C(ii), C(iii)
circle NO to the other TWO

c). Example of Reply Sheet

Assessment Symbol Question	A	B		C		
		(i)	(ii)	(i)	(ii)	(iii)
15(a)	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No
(b)	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No
(c)	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No
(d)	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No
(e)	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No	Yes/No

APPENDIX B5Brief aan Kandidate. Studentevraelys en Instruksies vir die Voltooiing daarvan.

Aan die student

In die aangehegde vraelys word u gevra vir u mening omtrent die vrae wat u nou net in die eksamenvraestel beantwoord het. Al wat u moet doen is om slegs JA of NEE in die gepaste plek op die Vraelysantwoordvel te omkring. Gebruik 'n potlood om die woorde te omkring.

U moet asseblief die vraestel naslaan terwyl u die vraelys beantwoord. Antwoord so sorgvuldig as u kan, maar u moet asseblief nie te veel tyd op elke vraag bestee nie.

Maak asseblief seker dat u die vraelys verstaan - veral die onderskeiding tussen elke aspek.

Volg asseblief die instruksies oor hoe u die vraelysantwoordvel moet invul en beskou die voorbeelde wat gegee word.

Baie dankie vir u help.



D. A. NORTON

VRAELYS WAT DEUR DIE LEERLINGE, ONMIDDELLIK NA HULLE
DIE EKSAMENVRAESTEL IN WISKUNDE HOER GRAAD VRAESTEL I
AFGELE HET, VOLTOOI MOET WORD

- LW. 1. Die woord "vraag" verwys in elke geval na die
vrae in die eksamenvraestel.
2. Omkring JA of NEE in die gepaste plek op die
Vraelysantwoordvel.

Meningsimbool

- A Het u hierdie soort vraag tevore al teëgekom?
(d.w.s. was die woorde wat gebruik is en die
probleemtype dieselfde of amper dieselfde as wat
u in klaswerkvrae teëgekom het?)
- B As u JA vir A geantwoord het:
i) Het u goed geleer hoe om hierdie vraag te
beantwoord?
ii) Het u die uitwerking maklik gevind?
- C As u NEE vir A geantwoord het, m.a.w. die vraag
was 'n soort wat u nog nie tevore uitgewerk het nie:
i) Was dit maklik om die verbinding tussen u kennis
en die nuwe situasie in te sien?
ii) Het u gevind dat u die vraag sorgvuldig moes
ontleed, en dat dit net was na u baie nagedink
het, dat u al u kennis kon byeenbring om die
vraag te beantwoord, alhoewel u sekere aspekte,
wat in die bewerking gebruik moes word, gou
herken het?
iii) Het u gevind dat die vraag u iets heeltemal
anders gevra het as die soort vraag waaraan u
gewoond is?

Instruksies vir die Voltooing van die Vraelysantwoordvel

1. Vul u studentenommer in.
 2. In elke ry:
 - a) As u JA vir A omkring het, antwoord B.
As u NEE vir A omkring het, antwoord C.
- LW Omkring-in OF B OF C, NIE beide nie.
- b) IN B omkring in beide B(i) en B(ii)
In C omkring JA vir net EEN van C(i), C(ii),
C(iii)
omkring NEE vir die ander TWEE.

c) Voorbeeld van Antwoordvel

Mening sim- bool Vraag	A	B		C		
		(i)	(ii)	(i)	(ii)	(iii)
15(a)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(b)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(c)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(d)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(e)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee

APPENDIX B6Vraelysantwoordvel

VRAELYSANTWOORDVEL 1

Studentenommer _____

Mening Vraag	A	B		C		
		(i)	(ii)	(i)	(ii)	(iii)
1(a)(i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(iii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(b)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(c)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(d)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(e)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
2(a)(i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(iii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(iv)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(v)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(b)(i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(iii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(iv)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(v)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(vi)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
3(a)(i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(b)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(c)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
4(a)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(b)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(c)(i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee

VRAELYSANTWOORDVEL 2

Studentenommer _____

Mening Vraag	A	B		C		
		(i)	(ii)	(i)	(ii)	(iii)
5(a)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(b)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(c)(i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(iii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
6(a)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(b)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(c)(i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
7(a)(i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(b)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(c)(i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(d)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
8 (i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
9(a)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(b)(i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
9 (i)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(ii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(iii)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(iv)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee
(v)	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee	Ja/Nee

APPENDIX C

Table of Specifications for the Examination Paper
given in Appendices A1 and A2

This table was drawn up immediately after the examination was set.

The only modification to the original is that questions listed separately under AS have been combined.

The figures next to the questions represent the number of marks allocated to each question.

Process Con- tent	K	CA	AS	I	Total Marks
Rela- tions and Func- tions	1(a)(i) 2 (ii) 2 (iii) 2 2a (i) 5 (iii) 1 2b(iv) 1 9b(ii) 3	2a(ii) 7 (v) 4 b(i) 3 (ii) 2 (iii) 5 (v) 2 8 (ii) 8 9b(i) 7	1c 5 2a(iv) 4 2b(vi) 2 8 (i) 11		76
Equa- tions and Inequal- ities	4a 5	3a(i) 5 b 7 9a 4	3a(ii) 3 4b 10 4c(i) 5	3c 7 4c(ii) 3	49
Indices and Logar- ithms	1b 3 d 2 5a 3 6c(i) 2	1e 4 5b 5 5c(i) 3 (ii) 5 6a 3 6b 7 6c(ii) 7	5c(iii) 7		51
Sequences and Series	7a(i) 1	7b 5 7c(i) 6	7a(ii) 2 c(ii) 2 d 8		24
Mathe- matical Induc- tion	9(iv) 1	9(ii) 6	9(i) 3 (iii) 2 (v) 2		14
Total Marks	33	105	66	10	214

APPENDIX D1Number of Pupils per Quintile for the Given Examination Questions

<u>Questions</u>	<u>Q1</u>	<u>Q2</u>	<u>Q3</u>	<u>Q4</u>	<u>Q5</u>	<u>Total</u>
1a - 3b; 4a - 5b } 7a(i) - 8(ii) }	52	52	50	52	52	258
3c	48	48	47	48	48	239
5c(i) - 6c(ii)	41	41	41	41	41	205
9a and 9b(ii)	36	36	34	36	36	178
9b(i)	32	32	31	32	32	159
9(i) - 9(v)	20	20	20	20	20	100

APPENDIX D2

Mean Percentage Score and Data on Teachers' Classifications including Resultant Classifications

Questions (N = 51)	Mean % Score	Classifications(N=14)				Resultant Classification
		K	CA	AS	I	
1(a)(i)	53	10	1	2	1	K
(ii)	56	10	-	4	-	K
(iii)	58	8	2	4	-	K
(b)	63	10	2	-	2	K
(c)	73	2	9	3	-	CA
(d)	48	3	3	6	2	Split AS
(e)	55	9	3	2	-	K
2(a)(i)	77	14	-	-	-	K
(ii)	55	8	6	-	-	K
(iii)	85	11	3	-	-	K
(iv)	58	2	6	6	6	CA/AS
(v)	40	5	9	-	-	CA
2(b)(i)	71	10	2	2	-	K
(ii)	45	3	9	1	1	CA
(iii)	48	9	4	-	1	K
(iv)	31	12	1	-	1	K
(v)	80	13	1	-	-	K
(vi)	10	1	6	5	2	Split CA
3(a)(i)	70	9	5	-	-	K
(ii)	37	8	4	2	-	K
(b)	32	2	9	1	2	CA
(c)	13	-	-	5	8	I
4(a)	56	9	5	-	-	K
(b)	31	3	9	2	-	CA
(c)(i)	26	-	2	10	2	AS
(ii)	5	-	1	5	8	I
5(a)	54	14	-	-	-	K
(b)	80	8	6	-	-	K
(c)(i)	79	8	6	-	-	K
(ii)	56	5	8	1	-	CA
(iii)	51	3	7	4	-	Split CA
6(a)	29	2	11	-	1	CA
(b)	55	8	6	-	-	K
(c)(i)	57	11	2	1	-	K
(ii)	38	4	9	1	-	CA
7(a)(i)	46	12	1	1	-	K
(ii)	17	4	2	7	1	Split AS
(b)	52	9	4	-	1	K
(c)(i)	27	4	8	2	-	CA
(ii)	40	9	3	2	-	K
(d)	20	1	7	5	1	Split CA
8(i)	41	1	2	9	2	Split AS
(ii)	16	1	3	8	2	AS
1)9(a)	38	3	5	3	-	Split CA

Questions (N = 51)	Mean % Score	Classifications(N=14)				Resultant Classification
		K	CA	AS	I	
9(b)(i)	13	1	5	3	2	Split CA
(ii)	23	2	8	1	-	CA
2)9(i)	29	2	3	3	-	CA/AS
(ii)	35	4	4	-	-	K/CA
(iii)	7	1	3	4	-	Split AS
(iv)	35	3	2	3	-	K/AS
(v)	10	1	3	4	-	Split AS

1) Only 11 teachers responded to this question

2) Only 8 teachers responded to this question

APPENDIX D3

Data on Pupils' Classification of Questions including
Resultant Classifications

Questions (N = 51)	Pupils' Classifications				Resultant Classification
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>	
1(a)(i)	134	54	27	39	K
(ii)	103	61	30	56	Split K
(iii)	101	60	34	56	Split K
(b)	170	55	10	10	K
(c)	104	48	53	42	Split K
(d)	41	34	85	92	Split K
(e)	128	56	13	51	K
2(a)(i)	180	54	6	9	K
(ii)	124	97	9	18	Split K
(iii)	171	30	25	24	K
(iv)	59	45	53	91	Split I
(v)	130	79	9	30	K
(b)(i)	164	50	19	16	K
(ii)	99	85	27	38	Split K
(iii)	105	82	18	45	Split K
(iv)	108	89	20	31	Split K
(v)	199	29	5	12	K
(vi)	21	70	50	107	Split I
3(a)(i)	159	75	8	4	K
(ii)	116	78	18	34	Split K
(b)	120	89	15	22	Split K
(c)	4	45	60	123	I
4(a)	166	38	18	27	K
(b)	62	103	39	44	Split CA
(c)(i)	27	77	64	75	Split CA
(ii)	16	30	65	139	I
5(a)	156	68	8	19	K
(b)	187	46	8	2	K
(c)(i)	143	40	4	5	K
(ii)	96	71	20	10	Split K
(iii)	103	56	26	9	K
6(a)	52	99	31	16	Split CA
(b)	101	65	18	14	K
(c)(i)	89	63	25	15	Split K
(ii)	68	98	10	13	CA
7(a)(i)	134	42	49	23	K
(ii)	87	50	50	58	Split K
(b)	100	88	15	44	Split K
(c)(i)	37	107	43	58	Split CA
(ii)	81	69	23	70	Split K
(d)	27	93	42	77	Split CA
8(i)	19	23	90	111	Split I
(ii)	31	40	48	122	I
9(a)	61	41	23	29	Split K
(b)(i)	26	30	49	37	Split AS
(ii)	32	29	29	58	Split I

Questions (N = 51)	Pupils' Classifications				Resultant Classification
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>	
9(i)	29	38	12	13	Split CA
(ii)	36	48	4	4	CA
(iii)	13	22	16	36	Split I
(iv)	26	24	13	22	Split K
(v)	12	53	8	11	CA

APPENDIX D4

Pupils Resultant Classifications by Quintile

These classifications were obtained in the same way as the overall resultant classifications given in Appendix D2 and D3.

Questions (N = 51)	Pupils' Resultant Classifications				
	Q1	Q2	Q3	Q4	Q5
1a(i)	K	K	K	Split K	Split CA
(ii)	K	K	Split K	Split CA	Split I
(iii)	K	K	K/CA	CA/I	Split I
b	K	K	K	K	Split K
c	Split K	Split K	Split K	Split K	Split I
d	AS	Split AS	Split I	Split I	Split I
e	K	K	K	Split K	Split I
2a(i)	K	K	K	K	Split K
(ii)	K	K	Split K	CA	CA
(iii)	K	K	K	K	Split K
(iv)	Split K	Split I	Split I	Split I	I
(v)	K	K	Split K	Split I	Split K
b(i)	K	K	K	K	K
(ii)	Split K	Split K	Split K	Split CA	Split I
(iii)	K	Split K	Split K	Split CA	Split I
(iv)	K	K	CA	Split AS	Split CA
(v)	K	K	K	K	K
(vi)	Split AS	Split CA	Split I	I	I
3a(i)	K	K	K	Split CA	Split CA
(ii)	K	K	Split CA	CA	Split CA
b	K	K	K	CA	Split CA
c	Split AS	I	Split I	I	I
4a	K	K	K	Split K	K
b	Split K	Split CA	Split CA	CA	Split CA
c(i)	Split AS	Split I	Split CA	Split CA	Split I
(ii)	I	I	I	Split I	I
5a	K	K	K	K	Split CA
b	K	K	K	K	Split K
c(i)	K	K	K	K	K
(ii)	K	K	K	Split K	Split CA
(iii)	K	K	K	Split K	Split CA
6a	Split CA	Split CA	Split CA	CA	CA
b	K	K	K	Split CA	CA
c(i)	K	K	Split K	Split CA	CA
(ii)	K	Split CA	CA	CA	CA
7a(i)	K	K	Split K	Split K	K
(ii)	Split K	Split K	CA/I	Split K	Split K
b	K	K	Split CA	CA	Split I
c(i)	K/CA	Split CA	CA	Split CA	Split CA
(ii)	K	Split K	Split CA	Split I	Split I
d	Split I	Split CA	CA	Split CA	Split I
8(i)	AS	Split I	I	I	I

Questions (N = 51)	Pupils' Resultant Classifications				
	Q1	Q2	Q3	Q4	Q5
8(ii)	Split K	Split I	I	I	I
9a	K	Split K	Split CA	Split CA	K/I
b(i)	Split AS	Split AS	Split AS	Split I	Split I
(ii)	Split K	Split K	Split I	I	I
9(i)	K/CA/I	CA	Split K	CA	Split K
(ii)	K	Split CA	CA	CA	CA
(iii)	Split I	Split I	Split I	Split I	Split I
(iv)	K	K	Split I	Split CA	Split I
(v)	Split CA	CA	CA	CA	CA

APPENDIX D5

Pupil Classification by Question Divided according to Quintiles

Question 1a(i)

Resultant Classification : Teacher K
Pupil K

<u>Group</u>	<u>Classifications</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	37	5	6	2	50	77
Q2	32	7	6	6	51	60
Q3	31	10	4	5	50	56
Q4	21	14	5	11	51	39
Q5	13	18	6	15	52	35
Totals	134	54	27	39	254	53

Overall difference : Significant ($P = 0,001$)

Differences between Quintiles (% is difference in Mean % Score)

<u>Significant ($P = 0,05$ at least)</u>		<u>Not Significant</u>	
Q1 - Q4	38%	Q1 - Q2	17%
Q1 - Q5	42%	Q1 - Q3	21%
Q2 - Q5	25%	Q2 - Q3	4%
Q3 - Q5	21%	Q2 - Q4	21%
		Q3 - Q4	17%
		Q4 - Q5	4%

APPENDIX D5 (contd)

Question 1a(ii)

Resultant Classification : Teachers : K

Pupils : Split K

<u>Group</u>	<u>Classifications</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	35	6	6	3	50	82
Q2	31	9	4	6	50	73
Q3	19	13	6	12	50	60
Q4	8	17	10	14	49	34
Q5	10	16	4	21	51	34
Totals	103	61	30	56	250	56

Overall Difference : Significant (P = 0,001)

Differences between Quintiles (% is difference in Mean % Score)

<u>Significant (P = 0,05 at least)</u>		<u>Not Significant</u>	
Q1 - Q3	22%	Q1 - Q2	9%
Q1 - Q4	48%	Q2 - Q3	13%
Q1 - Q5	48%	Q3 - Q4	26%
Q2 - Q4	39%	Q3 - Q5	26%
Q2 - Q5	39%	Q4 - Q5	0%

APPENDIX D5 (contd)

Question 1a(iii)

Resultant Classification : Teachers : K
 Pupils : Split K

<u>Group</u>	<u>Classifications</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	35	6	6	3	50	84
Q2	30	9	5	6	50	77
Q3	16	16	7	11	50	53
Q4	8	16	10	16	50	39
Q5	12	13	6	20	51	36
Totals	101	60	34	56	251	58

Overall Difference : Significant (P =0,001)

Differences between Quintiles (% is difference in Mean
 % Score)

<u>Significant (P = 0,05 at least)</u>		<u>Not Significant</u>	
Q1 - Q3	31%	Q1 - Q2	7%
Q1 - Q4	45%	Q3 - Q4	14%
Q1 - Q5	48%	Q3 - Q5	17%
Q2 - Q3	24%	Q4 - Q5	3%
Q2 - Q4	38%		
Q2 - Q5	41%		

APPENDIX D5(contd)

Question 1(c)

Resultant Classification : Teachers : CA
 Pupils : Split K

<u>Group</u>	<u>Classifications</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	25	9	12	4	50	87
Q2	25	9	12	4	50	85
Q3	20	12	12	6	50	81
Q4	21	9	8	11	49	67
Q5	13	9	9	17	48	48
Totals	104	48	53	42	247	73

Overall Difference : Significant (P = 0,05)

Differences between Quintiles (% is difference in Mean % Score)

Significant
(P=0,05 at least)Not Significant

Q1 - Q5 39%

Q1 - Q2 2% (Distribution : the same)

Q2 - Q5 37%

Q1 - Q3 6%

Q1 - Q4 20%

Q2 - Q3 4%

Q2 - Q4 18%

Q3 - Q4 14%

Q3 - Q5 37%

Q4 - Q5 19%

APPENDIX D5(contd)

Question 1d

Resultant Classification : Teachers : Split AS
 Pupils : Split I

<u>Group</u>	<u>Classifications</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	11	3	26	11	51	63
Q2	11	5	22	12	50	56
Q3	10	5	14	21	50	46
Q4	5	9	13	24	51	47
Q5	4	12	10	24	50	20
Totals	41	34	85	92	252	48

Overall Difference : Significant (P = 0,001)

Differences between Quintiles (% is difference in Mean
 % Score)

<u>Significant (P = 0,05 at least)</u>		<u>Not Significant</u>	
Q1 - Q4	16%	Q1 - Q2	7%
Q1 - Q5	33%	Q1 - Q3	17%
Q2 - Q4	9%	Q2 - Q3	10%
Q2 - Q5	26%	Q3 - Q4	1%
		Q3 - Q5	16%
		Q4 - Q5	17%

APPENDIX D5(contd)

Question 2a(iv)

Resultant Classification : Teachers : CA/AS
 Pupils : *Split I

<u>Group</u>	<u>Classifications</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	21	8	17	4	50	65
Q2	17	4	9	20	50	36
Q3	9	9	9	23	50	19
Q4	8	14	9	17	48	18
Q5	4	10	9	27	50	7
Total	59	45	53	91	248	29

Overall Difference : Significant (P = 0,001)

Differences between Quintiles (% is difference in Mean % Score)

<u>Significant (P = 0,05 at least)</u>		<u>Not Significant</u>	
Q1 - Q2	29%	Q2 - Q3	17%
Q1 - Q3	46%	Q3 - Q4	1%
Q1 - Q4	47%	Q3 - Q5	12%
Q1 - Q5	58%	Q4 - Q5	11%
Q2 - Q4	18%		
Q2 - Q5	29%		

APPENDIX D5(contd)

Question 2b(i)

Resultant Classification : Teachers : K
 Pupils : K

<u>Group</u>	<u>Classification</u>			<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS/I</u>		
Q1	41	4	5	50	87
Q2	36	11	3	50	87
Q3	31	12	7	50	75
Q4	30	12	7	49	66
Q5	26	11	13	50	38
Total	164	50	35	249	71

Overall Difference : Significant $P = 0,05$

Differences between Quintiles (% is difference in Mean % Score)

<u>Significant</u> <u>($P=0,05$ at least)</u>		<u>Not Significant</u>	
Q1 - Q5	49%	Q1 - Q2	0%
Q2 - Q5	49%	Q1 - Q3	12%
		Q1 - Q4	21%
		Q2 - Q3	12%
		Q2 - Q4	21%
		Q3 - Q4	9% (significantly the same)
		Q3 - Q5	37%
		Q4 - Q5	28%

APPENDIX D5(contd)

Question 2b(ii)

Resultant Classification : Teachers : CA
 Pupils : Split K

<u>Group</u>	<u>Classification</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	25	19	6	0	50	72
Q2	25	19	2	4	50	50
Q3	22	18	5	5	50	50
Q4	15	19	6	9	49	29
Q5	12	10	8	19	49	23
Total	99	85	27	37	248	45

Overall Difference : Significant ($P = 0,001$)

Differences between Quintiles (% is difference in Mean % Score)

<u>Significant ($P = 0,05$ at least)</u>		<u>Not Significant</u>	
Q1 - Q4	43%	Q1 - Q2	22%
Q1 - Q5	49%	Q1 - Q3	22%
Q2 - Q5	27%	Q2 - Q3	0%
Q3 - Q5	27%	Q2 - Q4	21%
		Q3 - Q4	21%
		Q4 - Q5	6%

APPENDIX D5 (contd)

Question 4b

Resultant Classification : Teachers : CA
 Pupils : Split CA

<u>Group</u>	<u>Classification</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	23	12	11	3	49	61
Q2	15	19	10	7	51	40
Q3	15	21	4	10	50	31
Q4	3	27	8	11	49	15
Q5	6	24	6	13	49	9
Total	62	103	39	44	248	31

Overall Difference : Significant (P = 0,001)

Differences between Quintiles (% is difference in Mean % Score)

<u>Significant (P = 0,05 at least)</u>		<u>Not Significant</u>	
Q1 - Q3	30%	Q1 - Q2	21%
Q1 - Q4	46%	Q2 - Q3	9%
Q1 - Q5	52%	Q2 - Q5	31%
Q2 - Q4	25%	Q3 - Q5	22%
Q3 - Q4	16%	Q4 - Q5	6%

APPENDIX D5 (Contd)

Question 4c(i)

Resultant Classification : Teachers : AS
 Pupils : Split CA

<u>Group</u>	<u>Classification</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	5	7	20	16	48	39
Q2	8	14	14	15	51	32
Q3	8	19	13	10	50	26
Q4	3	19	12	14	48	21
Q5	3	19	5	20	47	12
Total	27	78	64	75	244	26

Overall Difference : Significant ($P = 0,025$)

Differences between Quintiles (% is difference in Mean % Score)

<u>Significant ($P = 0,05$ at least)</u>		<u>Not Significant</u>	
Q1 - Q3	13%	Q1 - Q2	7%
Q1 - Q5	27%	Q1 - Q4	18%
Q3 - Q5	14%	Q2 - Q3	6%
		Q2 - Q4	11%
		Q2 - Q5	20%
		Q3 - Q4	5%
		Q4 - Q5	9%

APPENDIX D5 (contd)

Question 7a(ii)

Resultant Classification : Teachers : Split AS
 Pupils : Split K

<u>Group</u>	<u>Classification</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	25	4	13	8	50	63
Q2	19	12	9	11	51	11
Q3	10	14	11	14	49	5
Q4	17	9	9	10	45	0
Q5	16	11	8	15	50	3
Total	87	50	50	58	245	17

Overall Difference : Not Significant

Differences between Quintiles (% is difference in Mean
 % Score)

<u>Significant (P = 0,05 at least</u>		<u>Not Significant</u>	
Q1 - Q3	58%	Q1 - Q2	52%
Q1 - Q5	60%	Q1 - Q4	63%
		Q2 - Q3	6%
		Q2 - Q4	11%
		Q2 - Q5	8%
		Q3 - Q5	2%
		Q3 - Q4	5%
		Q4 - Q5	3%

APPENDIX D5 (contd)

Question 7c(i)

Resultant Classification : Teachers : CA
 Pupils : Split CA

<u>Group</u>	<u>Classification</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	14	14	13	9	50	56
Q2	7	23	11	9	50	32
Q3	4	30	5	10	49	20
Q4	4	21	7	16	48	14
Q5	8	19	7	14	48	8
Total	37	107	43	58	245	27

Overall Difference : Significant $P = 0,025$

Differences between Quintiles

<u>Significant (P = 0,05 at least)</u>		<u>Not Significant</u>	
Q1 - Q3	36%	Q1 - Q2	24%
Q1 - Q4	42%	Q1 - Q5	48%
		Q2 - Q3	12%
		Q2 - Q4	18%
		Q2 - Q5	24%
		Q3 - Q4	6%
		Q3 - Q5	12%
		Q4 - Q5	6%

APPENDIX D5 (contd)

Question 8(ii)

Resultant Classification : Teachers : AS
 Pupils : I

<u>Group</u>	<u>Classification</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	20	5	18	8	51	51
Q2	6	14	9	18	47	15
Q3	3	8	7	30	48	5
Q4	2	8	5	35	50	5
Q5	0	5	9	31	45	0
Total	31	40	48	122	241	16

Overall Difference : Significant (P = 0,001)

Differences between Quintiles (% is difference in Mean
 % Score)

<u>Significant (P = 0,05 at least)</u>		<u>Not Significant</u>	
Q1 - Q2	36%	Q3 - Q4	5%
Q1 - Q3	46%	Q3 - Q5	5%
Q1 - Q4	46%	Q4 - Q5	5%
Q1 - Q5	51%		
Q2 - Q3	10%		
Q2 - Q4	10%		
Q2 - Q5	15%		

APPENDIX D5 (contd)

Question 9b(ii)

Resultant Classification : Teachers : CA
 Pupils : Split I

<u>Group</u>	<u>Classification</u>				<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS</u>	<u>I</u>		
Q1	14	5	11	4	34	65
Q2	10	8	7	7	32	21
Q3	4	6	5	11	26	21
Q4	2	6	3	16	27	0
Q5	2	4	3	20	29	3
Total	32	29	29	58	148	23

Overall Difference : Significant (P = 0,001)

Differences between Quintiles (% is difference in Mean
 % Score)

<u>Significant (P = 0,05 at least)</u>		<u>Not Significant</u>	
Q1 - Q3	44%	Q1 - Q2	44%
Q1 - Q4	65%	Q2 - Q3	0%
Q1 - Q5	62%	Q3 - Q4	21%
Q2 - Q4	21%	Q3 - Q5	18%
Q2 - Q5	18%	Q4 - Q5	3%

APPENDIX D5(contd)

Question 2a(iii)

Resultant Classification : Teachers : K
 Pupils : K

<u>Group</u>	<u>Classification</u>			<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS/I</u>		
Q1	40	3	7	50	96
Q2	45	2	4	51	96
Q3	39	4	7	50	94
Q4	27	12	10	49	76
Q5	20	9	21	50	62
Totals	171	30	49	250	85

Overall Difference : Significant (P = 0,001)

Differences between Quintiles (% is difference in Mean
 % Score)

<u>Significant</u> <u>(P=0,05 at least)</u>		<u>Not Significant</u>	
Q1 - Q4	20%	Q1 - Q2	0%
Q1 - Q5	34%	Q1 - Q3	2% (significantly the same)
Q2 - Q4	20%	Q2 - Q3	2%
Q2 - Q5	34%	Q4 - Q5	14%
Q3 - Q4	18%		
Q3 - Q5	32%		

APPENDIX D5 (contd)

Question 3b

Resultant Classification : Teachers : CA
 Pupils : Split K

<u>Group</u>	<u>Classifications</u>			<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS/I</u>		
Q1	27	18	5	50	54
Q2	34	10	5	49	34
Q3	28	16	5	49	38
Q4	15	26	8	49	21
Q5	16	19	14	49	13
Total	120	89	37	246	32

Overall Difference : Significant ($P = 0,001$)

Differences between Quintiles (% is difference in Mean % Score)

<u>Significant ($P = 0,05$ at least)</u>		<u>Not Significant</u>	
Q1 - Q5	41%	Q1 - Q2	20%
Q2 - Q4	13%	Q1 - Q3	16%
Q2 - Q5	21%	Q1 - Q4	33%
Q3 - Q4	17%	Q2 - Q3	4%
Q3 - Q5	25%	Q4 - Q5	8%

APPENDIX D5 (contd)

Question 6a

Resultant Classification : Teachers : CA
 Pupils : Split CA

<u>Group</u>	<u>Classifications</u>			<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS/I</u>		
Q1	13	18	9	40	61
Q2	12	19	10	41	47
Q3	11	15	13	39	15
Q4	7	24	8	39	15
Q5	9	23	7	39	7
Total	<u>52</u>	<u>99</u>	<u>47</u>	<u>198</u>	<u>29</u>

Overall Difference : Not Significant

Differences between Quintiles (% is difference in Mean % Score)

Significant
(P= 0,05 at least)

nil

Not
SignificantQ1 - Q2 14% (significantly
the same)

Q1 - Q3 46%

Q1 - Q4 46%

Q1 - Q5 54%

Q2 - Q3 32%

Q2 - Q4 32%

Q2 - Q5 40%

Q3 - Q4 0%

Q3 - Q5 8%

Q3 - Q5 8%

APPENDIX D5 (contd)

Question 6b

Resultant Classification : Teachers : K
 Pupils : K

<u>Group</u>	<u>Classifications</u>			<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS/I</u>		
Q1	30	5	5	40	84
Q2	24	11	5	40	76
Q3	25	10	5	40	63
Q4	12	19	8	39	33
Q5	10	20	9	39	18
Total	101	65	32	198	55

Overall Difference : Significant (P = 0,001)

Differences between Quintiles (% is difference in Mean
 % Score)

<u>Significant</u> (P=0,05 at least)		<u>Not</u> <u>Significant</u>	
Q1 - Q4	51%	Q1 - Q2	8%
Q1 - Q5	66%	Q1 - Q3	21%
Q2 - Q4	43%	Q2 - Q3	13% (significantly the same)
Q2 - Q5	58%	Q4 - Q5	15%
Q3 - Q4	30%		
Q3 - Q5	45%		

APPENDIX D5 (contd)

Question 7b

Resultant Classification : Teachers : K
 Pupils : Split K

<u>Group</u>	<u>Classifications</u>			<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS/I</u>		
Q1	29	9	13	51	70
Q2	28	15	7	50	77
Q3	17	23	9	49	55
Q4	14	28	7	49	41
Q5	12	13	23	48	19
Total	100	88	59	247	53

Overall Difference : Significant (P = 0,001)

Differences between Quintiles (% is difference in Mean
 % Score)

<u>Significant (P = 0,05 at least)</u>		<u>Not Significant</u>	
Q1 - Q3	15%	Q1 - Q2	7%
Q1 - Q4	29%	Q2 - Q3	22%
Q1 - Q5	51%	Q3 - Q4	14%
Q2 - Q4	26%		
Q2 - Q5	58%		
Q3 - Q5	36%		
Q4 - Q5	22%		

APPENDIX D5 (contd)

Question 7c(ii)

Resultant Classification : Teachers : K

Pupils : Split K

<u>Group</u>	<u>Classifications</u>			<u>Total</u>	<u>Average %</u>
	<u>K</u>	<u>CA</u>	<u>AS/I</u>		
Q1	31	11	9	51	74
Q2	21	15	13	49	48
Q3	12	18	18	48	30
Q4	12	13	24	49	29
Q5	5	12	29	46	17
Total	81	69	93	243	40

Overall Difference : Significant ($P = 0,001$)

Differences: between Quintiles (% is difference in Mean % Score)

<u>Significant ($P = 0,05$ at least)</u>			<u>Not Significant</u>	
Q1 - Q3	44%		Q1 - Q2	26%
Q1 - Q4	45%		Q2 - Q3	18%
Q1 - Q5	57%		Q2 - Q4	19%
Q2 - Q5	31%		Q3 - Q4	1%
Q3 - Q5	13%		Q4 - Q5	12%

APPENDIX D5 (contd)

Question 2b(vi)

Resultant Classification : Teachers : Split CA
 Pupils : Split I

<u>Group</u>	<u>Classifications</u>			<u>Total</u>	<u>Average %</u>
	<u>K/CA</u>	<u>AS</u>	<u>I</u>		
Q1	21	16	14	51	33
Q2	24	10	16	50	7
Q3	19	6	25	50	5
Q4	12	12	25	49	3
Q5	15	6	27	48	3
Total	91	50	107	248	11

Overall Difference : Significant ($P = 0,025$)

Differences between Quintiles (% is difference in Mean % Score)

<u>Significant ($P = 0,05$ at least)</u>		<u>Not Significant</u>	
Q1 - Q3	28%	Q1 - Q2	26%
Q1 - Q4	30%	Q2 - Q3	2%
Q1 - Q5	30%	Q2 - Q5	4%
Q2 - Q4	4%	Q3 - Q4	2%
		Q3 - Q5	2%
		Q4 - Q5	0%

APPENDIX D5 (contd)

Question 3c

Resultant Classification : Teachers : I
 Pupils : I

<u>Group</u>	<u>Classifications</u>			<u>Total</u>	<u>Average %</u>
	<u>K/CA</u>	<u>AS</u>	<u>I</u>		
Q1	13	20	15	48	47
Q2	8	9	29	46	13
Q3	14	10	22	46	3
Q4	3	13	32	48	2
Q5	11	8	25	44	1
Total	49	60	123	232	13

Overall Difference : Significant ($P = 0,005$)

Differences between Quintiles (% is difference in Mean % Score)

<u>Significant ($P = 0,05$ at least)</u>		<u>Not Significant</u>	
Q1 - Q2	34%	Q1 - Q3	44%
Q1 - Q4	45%	Q2 - Q3	10%
Q1 - Q5	46%	Q2 - Q4	11%
Q3 - Q4	1%	Q2 - Q5	12%
Q4 - Q5	1%	Q3 - Q5	2%

APPENDIX D5 (contd)

Question 4c(ii)

Resultant Classification : Teachers : I
 Pupils : I

<u>Group</u>	<u>Classifications</u>			<u>Total</u>	<u>Average %</u>
	<u>K/CA</u>	<u>AS</u>	<u>I</u>		
Q1	7	16	27	50	17
Q2	8	15	28	51	5
Q3	12	8	30	50	0
Q4	8	19	24	51	1
Q5	11	7	30	48	0
Total	46	65	139	250	5

Overall Difference : Not Significant

Differences between Quintiles (% is difference in Mean
 % Score)

<u>Significant</u> (P = 0,05 at least)		<u>Not</u> <u>Significant</u>	
Q3 - Q4	1%	Q1 - Q2	12%
Q4 - Q5	1%	Q1 - Q3	17%
		Q1 - Q4	16%
		Q1 - Q5	17%
		Q2 - Q3	5%
		Q2 - Q4	4%
		Q2 - Q5	5%
		Q3 - Q5	0% (significantly the same)

APPENDIX D5 (contd)

Question 8(i)

Resultant Classification : Teachers : Split AS
 Pupils : Split I

<u>Group</u>	<u>Classifications</u>			<u>Total</u>	<u>Average %</u>
	<u>K/CA</u>	<u>AS</u>	<u>I</u>		
Q1	8	35	8	51	80
Q2	13	16	19	48	48
Q3	7	14	27	48	34
Q4	8	16	27	51	28
Q5	6	9	30	45	9
Total	42	90	111	243	41

Overall Difference : Significant ($P = 0,001$)

Differences between Quintiles (% is difference in Mean
 % Score)

<u>Significant ($P = 0,05$ at least)</u>		<u>Not Significant</u>	
Q1 - Q2	32%	Q2 - Q3	14%
Q1 - Q3	46%	Q2 - Q4	20%
Q1 - Q4	52%	Q3 - Q4	6%
Q1 - Q5	71%	Q3 - Q5	25%
Q2 - Q5	39%	Q4 - Q5	19%

APPENDIX D6

Pupils' Average Percentage Scores Based on their own
Classification for each Question

CLASSIFICATION OF QUESTION BY PUPILS

<u>Question</u>	<u>K</u>		<u>CA</u>		<u>AS</u>		<u>I</u>	
	<u>N</u>	<u>Av</u>	<u>N</u>	<u>Av</u>	<u>N</u>	<u>Av</u>	<u>N</u>	<u>Av</u>
1a(i)	134	66	54	42	27	52	39	27
(ii)	103	83	61	42	30	48	56	26
(iii)	101	85	60	38	34	59	56	29
b	170	81	55	27	10	20	10	25
c	104	83	48	90	53	76	42	51
d	41	79	34	47	85	54	92	30
e	128	80	56	39	13	40	51	11
2a(i)	180	84	54	65	6	63	9	31
(ii)	124	78	97	32	9	56	18	25
(iii)	171	94	30	63	25	80	24	50
(iv)	132	56	50	28	72	34	38	10
(v)	130	53	79	32	9	31	30	10
2b(i)	164	80	50	55	19	56	16	35
(ii)	99	56	85	45	27	37	38	21
(iii)	105	71	82	43	18	31	45	12
(iv)	108	52	89	20	20	5	31	3
(v)	199	85	29	66	5	50	12	54
(vi)	21	17	70	12	50	17	107	5
3a(i)	159	79	75	53	8	60	4	30
(ii)	116	56	78	20	18	33	34	12
b	120	39	89	28	15	24	22	16
c	4	29	45	16	60	20	123	8
4a	166	70	38	29	18	38	27	18
b	62	61	103	23	39	25	44	13
c(i)	27	39	78	23	64	30	75	22
(ii)	16	13	30	0	65	8	139	3
5a	156	71	68	32	8	21	19	7
b	187	83	46	67	8	83	2	50
c(i)	143	83	40	68	4	92	5	53
(ii)	96	83	71	31	20	39	10	16
(iii)	103	55	56	40	26	61	9	43
6a	52	46	99	22	31	33	16	15
b	101	75	65	30	18	52	14	31
c(i)	89	86	63	22	25	54	15	30
(ii)	68	70	98	24	10	16	13	13
7a(i)	134	51	42	43	49	45	23	30
(ii)	87	24	50	9	50	22	58	8
b	100	79	88	44	15	37	44	11
c(i)	37	47	107	24	43	31	58	16
(ii)	81	78	69	23	23	33	70	14
d	27	56	93	37	42	32	77	6
8(i)	19	69	23	34	90	51	111	29
(ii)	31	56	40	14	48	22	122	3
9a	61	54	41	32	23	49	29	2
b(i)	26	45	30	11	49	8	37	0

Question	<u>K</u>		<u>CA</u>		<u>AS</u>		<u>I</u>	
	<u>N</u>	<u>Av</u>	<u>N</u>	<u>Av</u>	<u>N</u>	<u>Av</u>	<u>N</u>	<u>Av</u>
9b(ii)	32	59	29	20	29	28	58	3
9(i)	29	33	38	32	12	25	13	18
(ii)	36	63	48	19	4	8	4	0
(iii)	13	31	22	0	16	6	36	4
(iv)	26	69	24	17	13	31	22	14
(v)	12	29	53	3	8	25	11	14
<u>N</u>	4630		3022		1463		2058	
Overall average		69,7		32,4		36,7		15,4

APPENDIX D7Pupils' Average Percentage Scores for each question
by Quintile

<u>Teachers Resultant Classification</u>	<u>Question</u>	<u>Average Percentage Scores</u>				
		<u>Q1</u>	<u>Q2</u>	<u>Q3</u>	<u>Q4</u>	<u>Q5</u>
K	1a(i)	77	60	56	39	35
	(ii)	82	73	60	34	33
	(iii)	84	77	53	39	35
	b	91	80	65	46	33
	e	92	76	56	38	11
	2a(i)	94	90	81	72	50
	(ii)	92	79	63	28	10
	(iii)	96	96	94	76	62
	2b(i)	87	87	75	66	38
	(iii)	84	62	52	32	9
	(iv)	71	42	26	12	2
	(v)	95	84	86	70	67
	3a(i)	90	80	79	58	40
	(ii)	73	49	40	13	9
	4a	79	62	56	45	37
	5a	78	67	63	46	16
	b	92	86	86	77	55
	c(i)	92	90	78	73	62
	6b	84	76	63	33	18
	6c(i)	81	68	53	45	33
	7a(i)	75	54	55	23	24
	b	70	77	55	41	19
	c(i)	74	48	30	29	16
	*9(ii)	72	39	39	11	12
	* (iv)	53	33	37	18	29
CA	1c	87	85	81	67	48
	*2a(iv)	65	36	19	18	7
	(v)	65	53	38	17	28
	2b(ii)	72	50	50	29	22
	(vi)	33	7	5	3	3
	3b	54	34	38	21	13
	4b	61	40	31	15	9
	5c(ii)	85	67	67	38	23
	(iii)	83	66	47	36	22
	6a	61	47	15	15	7
	c(ii)	65	56	29	24	21
	7c(i)	58	32	20	14	8
	d	51	17	14	8	3
	9a	78	39	27	22	17
	b(i)	36	24	4	2	0
	(ii)	65	21	21	0	1
	*9(i)	44	33	25	21	24
	* (ii)	72	39	39	11	12
AS	1d	63	56	46	47	30
	*2a(iv)	65	36	19	18	7
	4c(i)	39	32	26	21	12

<u>Teachers Resultant Classification</u>	<u>Question</u>	<u>Average Percentage Scores</u>				
		<u>Q1</u>	<u>Q2</u>	<u>Q3</u>	<u>Q4</u>	<u>Q5</u>
	7a(ii)	63	11	5	0	3
	8(i)	80	49	34	28	9
	(ii)	51	15	5	5	0
	*9(i)	44	33	25	21	24
	(iii)	8	13	3	8	7
	* (iv)	53	33	37	18	29
	(v)	16	18	3	15	0
I	3c	47	31	3	2	1
	4c(ii)	17	5	0	1	0

* Questions included under two resultant classifications